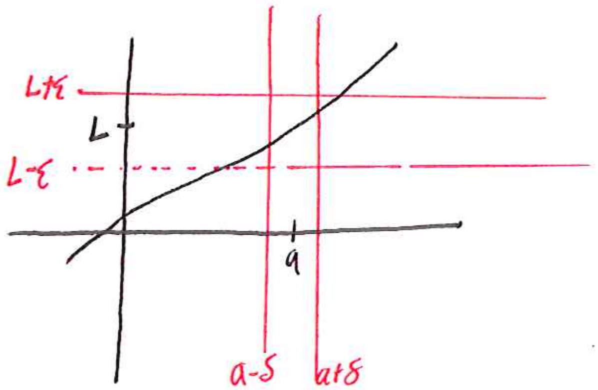
Lecture 3

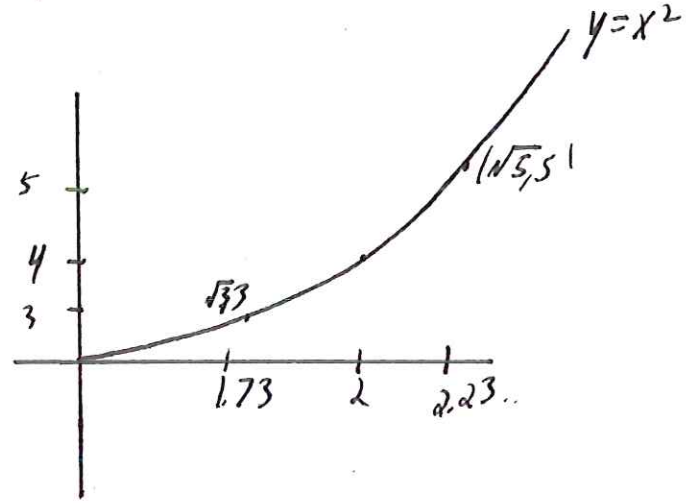
Review · Def of $\lim_{x \to a} f(x) = L$, $f(a)$ irrelevant, only values near a matter.



$\forall \varepsilon$, want $L-\varepsilon < f(x) < L+\varepsilon$

i.e. $|f(x)-L| < \varepsilon$

For any $\varepsilon > 0$ there exists $\delta > 0$ so that if $0<|x-a|<\delta$ then $|f(x)-L|<\varepsilon$. * Leave up *



Example $\lim_{x \to 2} x^2 = 4$

ε	δ
1	.23
.1	.02

$\sqrt{4.1} = 2.024\ldots$

$\sqrt{3.9} = 1.97\ldots$

Proof Let $\varepsilon > 0$ be given. Choose $\delta = \sqrt{4+\varepsilon} - 2$. Assume $2-\delta < x < 2+\delta$ and prove $|x^2-4| < \varepsilon$. * alternate proof !

Ex Prove that $\lim_{x \to 2} 6x-1 = 11$

Ex $\lim_{x \to 0} \frac{\sin x}{x} = 1$. Proof later...

Ex Prove that $\lim_{x\to 0} \sin(\pi/x)$ does not exist.

Ex $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2} x+2 = 4.$

↖ ok to cancel since $x=2$ is not relevant!

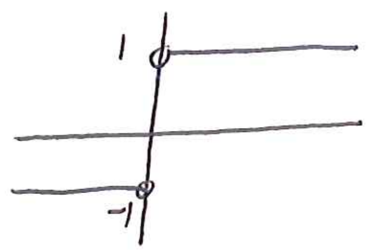
Interesting Example

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ \frac{1}{q} & \text{if } x = \frac{p}{q} \text{ in lowest terms} \end{cases}$$

When does $\lim_{x\to a} f(x)$ exist?

II. One-sided limits

Consider $f(x) = \frac{|x|}{x}$



so $\lim_{x\to 0} f(x)$ DNE.

Want to say $\lim_{x\to 0^+} f(x) = 1$ "as x approaches 0 from the right"

$\lim_{x\to 0^-} f(x) = -1$ "as x approaches 0 from the left".

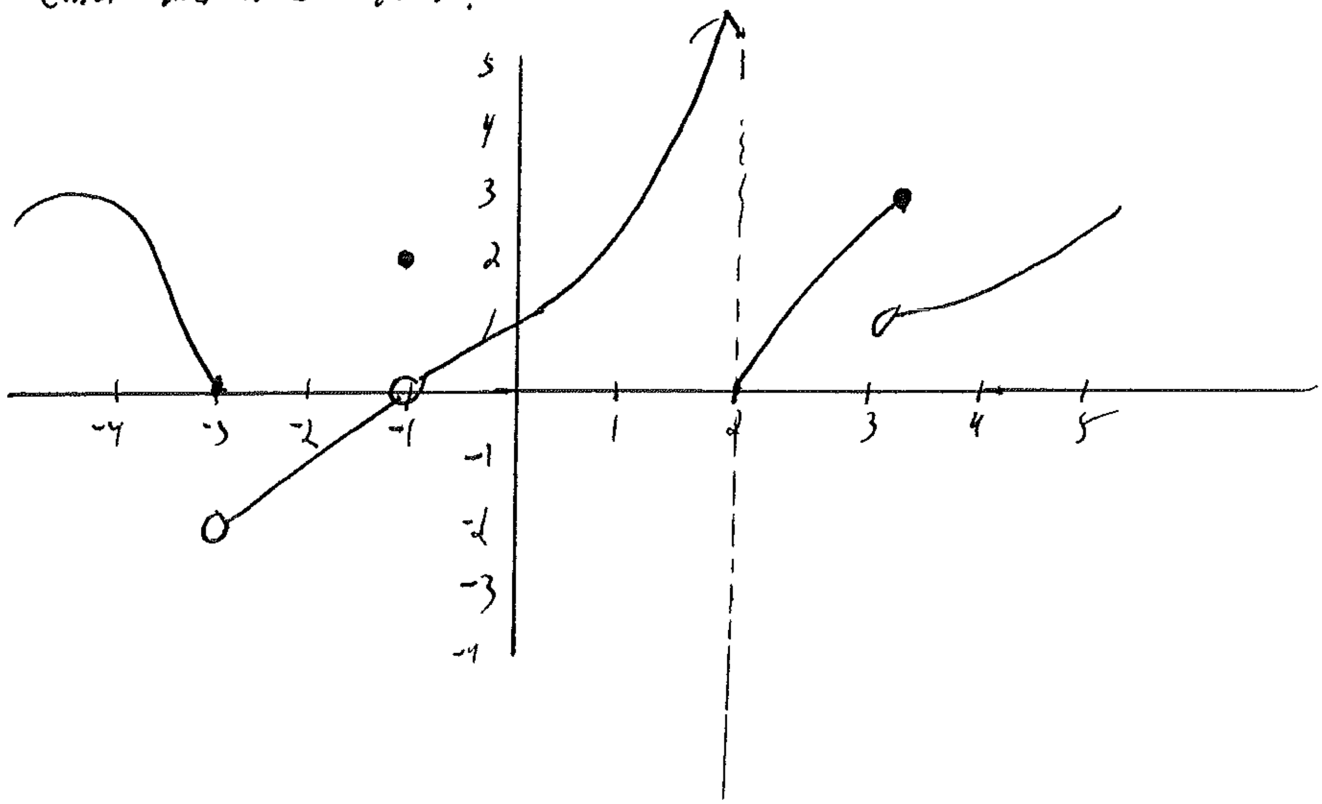
Def. Say $\lim_{x\to a^+} f(x) = L$ if for any $\varepsilon > 0$ there is a $\delta > 0$ so that if $\underline{0 < x-a < \delta}$ then $|f(x) - L| < \varepsilon$.

Say $\lim_{x\to a^-} f(x) = L$ if " " " " " " if $\underline{0 < a-x < \delta}$

<u>Easy Fact</u> $\lim_{x\to a} f(x)$ exists if & only if both $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$

exist and are equal.

<u>Example</u>



<u>Ex</u> $\lim_{x\to 0^+} \sqrt{x} = 0$

<u>Ex</u> Sketch function w/ given limits.

<u>III</u>. Infinite limits.

· Def

· EX