Lecture 25

Review

1. $f'(x) > 0$ on interval $\Rightarrow$ f is increasing on interval

2. $f'(x) < 0$ on interval $\Rightarrow$ f is decreasing on interval

Ex $f(x) = \frac{1}{2}x^4 - 4x^2 + 3$

$$f'(x) = 2x^3 - 8x = 2x(x^2-4) = 2x(x-2)(x+2)$$

Set $f' = 0$

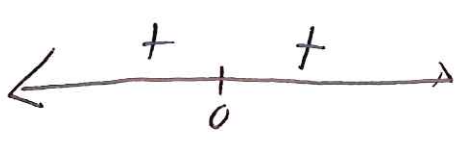
decreasing $(-\infty,-2) \cup (0,2)$
incr. $(-2,0) \cup (2,\infty)$

Ex $f(x) = \dfrac{e^x}{1-e^x}$

$$f'(x) = \frac{(1-e^x)e^x - e^x(-e^x)}{(1-e^x)^2} = \frac{1-e^{2x}+e^{2x}}{(1-e^x)^2} = \frac{1}{(1-e^x)^2}$$

Rmk $f'(x)$ can change signs it it $= 0$ or is undefined

$$1-e^x = 0 \Rightarrow x = 0$$



increasing $(-\infty,0) \cup (0,\infty)$

First Derivative Test

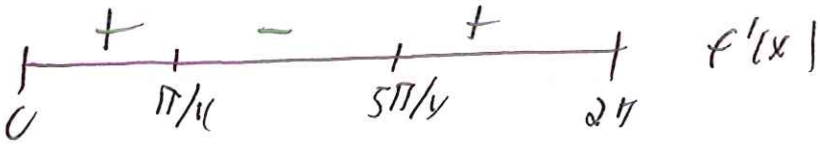
Suppose c is a critical # of $f(x)$

1. If f' changes pos. to neg at c then local max
2. " " " neg " pos " " " " min
3. Else neither

EX $f(x) = \sin x + \cos x$ $\quad 0 \le x \le 2\pi$. Find local max / min

$f'(x) = \cos x - \sin x$

$0 = f'(x) \Rightarrow \tan x = 1 \Rightarrow x = \pi/4,\ 5\pi/4$



local max at $(\pi/4, \sqrt{2})$ local min at $(5\pi/4, -\sqrt{2})$

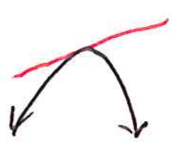
Q What does $f''(x)$ tell us about graph?

Concavity

concave up

"U" shape
graph lies above tangent

concave down



graph lies below tangents

f' is increasing

f' is decr.

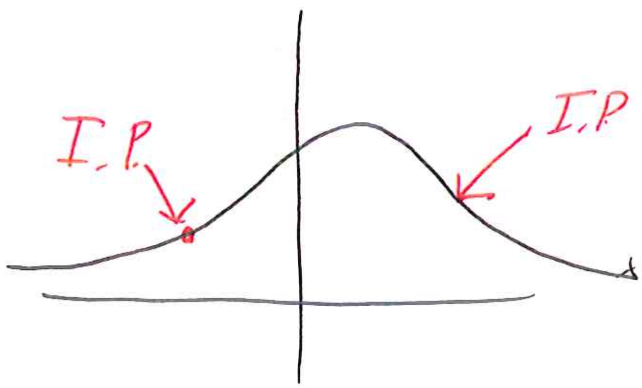
Concavity Test

1. If $f''(x)$ is >0 on interval then $f(x)$ is concave up on interval
2. If $f''(x)<0$ " " " " " " down

4 possibilities

f'	f''	
+	+	increasing & concave up
+	−	increasing & concave down.
−	+	decr & conc up
−	−	decr & conc do.

Def Point P on $y=f(x)$ is an <u>inflection point</u> if concavity changes there



Second Derivative Test

1. If $f'(c)=0$ and $f''(c)>0$ then local min at $x=c$ $\cup$
2. If $f'(c)=0$ and $f''(c)<0$ " " max. at $x=c$. $\cap$
3. If $f'(c)=0$ and $f''(c)=0$ No info Ex $y=x^4$, $y=-x^4$, $y=x^3$ all have $f'(0)=f''(0)=0$.

Ex $y = x^4 - 4x^3$. Do everything

Ex $y = x^2 \ln x$

Ex $y = \dfrac{x^2}{x-1}$. Classify local max/min w/ both tests

Ex $y = x^2 - x - \ln x$