Lecture 24

Mean Value Thm

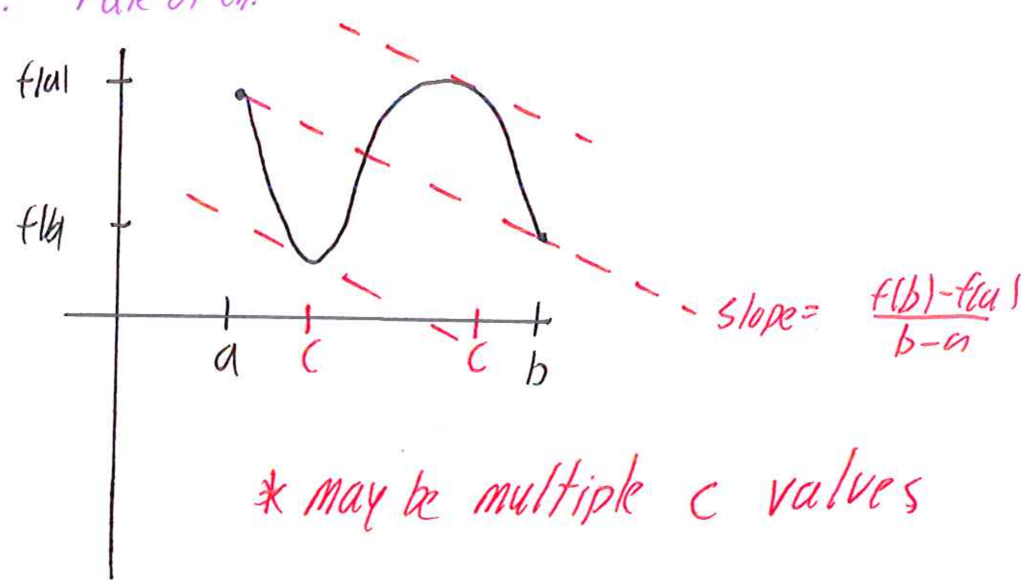
Informal: avg rate of change from $x=a$ to $x=b$
= instantaneous ROC at some $c \in (a,b)$

Thm Suppose $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b).

Then there exists a c in (a,b) with

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

inst. rate of ch.   avg ROC



* may be multiple c values

Ex $f(x) = 2x^2-3x+1$ on $[0,2]$

- Verify MVT applies
- Find all c

$$\frac{f(b)-f(a)}{b-a} = \frac{3-1}{2-0} = 1$$

$f'(x) = 4x-3$

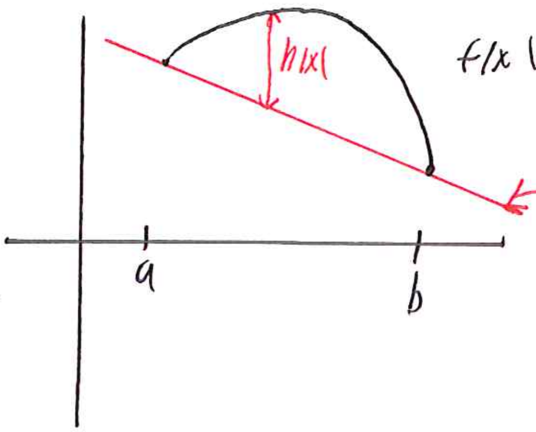
Set $=1$

$4x-3=1$

$x=1$

$$\boxed{c=1}$$

Proof of MVT



$h(x) =$ difference

$y - f(a) = \frac{f(b)-f(a)}{b-a}(x-a)$

$y = f(a) + \frac{f(b)-f(a)}{b-a}(x-a)$

Let $h(x) = f(x) - f(a) - \frac{f(b)-f(a)}{b-a}(x-a)$

$h(a) = h(b) = 0$. Apply Rolle's Thm to $h(x)$.

There is a c w/ $h'(c) = 0$

$$h'(x) = f'(x) - \frac{f(b)-f(a)}{b-o}$$

$$0 = f'(c) - \frac{f(b)-f(a)}{b-a}. \qquad //$$

Corollary 1 If $f'(x) = 0 \ \forall x \in (a,b)$ then f is constant on (a,b).

Corollary 2 Suppose $f'(x) = g'(x) \ \forall x \in (a,b)$.

Then $f(x) = g(x) + C$ for some C

Proof Apply Cor 1 to $f(x) - g(x)$.

Ex $f(x) = \frac{1}{(x-3)^2}$ on $[1,4]$. Show there is no c.
Why does this not contradict MVT?

Ex $f(x) = \ln x$ on $[1,4]$. Find c values.

Rmk MVT has many applications, we will just see a few

4.3 Curve Sketching

Def $f(x)$ is *increasing* on (a,b) if $a \le x_1 < x_2 \le b \Rightarrow f(x_1) < f(x_2)$
decreasing " " " " $f(x_1) > f(x_2)$!

Thm

1. If $f'(x) > 0$ on (a,b) then $f(x)$ is increasing on (a,b)
2. If $f'(x) < 0$ " " " " " decreasing "

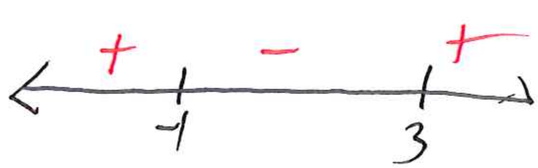
Proof Mean value Thm!

Ex $f(x) = x^3 - 3x^2 - 9x + 4$. Find intervals of increasing/dec.

A: $f'(x) = 3x^2 - 6x - 9$.

* By int. value thm, continuous function cannot change sign w/out being 0.

$0 = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x-3)(x+1)$



$f'(0) = -9$
$f'(4) = 15$
$f'(-2) = 15$

Increasing on $(-\infty, -1) \cup (3, \infty)$

Decreasing on $(-1, 3)$

Ex $f(x) = \sin x + \cos x$ on $[0, 2\pi]$ Find intervals of ↗ or ↘

$f'(x) = \cos x - \sin x$

$0 = \sin x - \cos x$

$1 = \tan x$

$\sin x = \cos x$

$x = \pi/4, 5\pi/4$

increasing $[0, \pi/4] \cup [5\pi/4, 2\pi]$

decreasing $(\pi/4, 5\pi/4)$

Question Suppose c is a critical value. Is $(c, f(c))$ a local max, min or neither?

First Derivative Test

1. If f' changes $+$ to $-$ at c, local max.
2. If f' changes $-$ to $+$ at c, local min
3. Other two cases, neither.