Lecture 18

Review Implicit diff

Ex Find tangent line to $2(x^2+y^2)^2 = 25(x^2-y^2)$ at $(3,1)$

$4(x^2+y^2)(2x+2yy') = 25(2x-2yy')$ easier to plug in 1st rather than solve for y'

$4\cdot(10)(6+2y') = 25(6-2y')$

$240 + 80y' = 150 - 50y'$ $\quad 130y' = -90$

$y' = -9/13$

$$\boxed{y-1 = -\tfrac{9}{13}(x-3)}$$

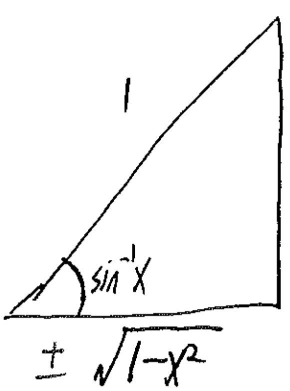
Derivatives of Inverse Trig functions

Recall $y = \sin^{-1}x$ means $\sin y = x$ and $-\pi/2 \le y \le \pi/2$

Differentiate implicity:

$\cos y \cdot y' = 1$

$y' = \dfrac{1}{\cos y} = \dfrac{1}{\cos(\sin^{-1}x)}$

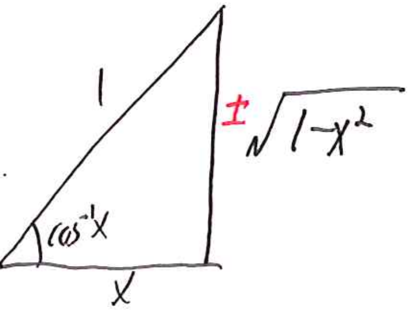


$\cos y$ is > 0 since $-\pi/2 \le y \le \pi/2$.

Conclude: $\boxed{\dfrac{d}{dx}\left(\sin^{-1}x\right) = \dfrac{1}{\sqrt{1-x^2}}}$

$y = \cos^{-1}x$ means $\cos y = x$ and $0 \le y \le \pi/2$

$$-\sin y \cdot y' = 1 \Rightarrow y' = \frac{-1}{\sin(\cos^{-1}x)}$$



$\sin y \ge 0$ since $0 \le y \le \pi/2$.

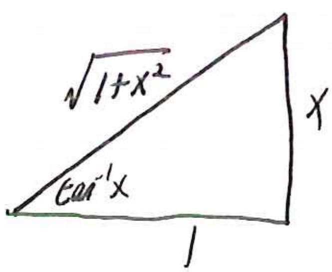
So $\sin(\cos^{-1}x) = \sqrt{1-x^2}$.

Conclude: $$\boxed{\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}}$$

$y = \tan^{-1}x$ means $\tan y = x$ and $-\pi/2 < y < \pi/2$

$$\sec^2 y \, \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2(\tan^{-1}x)} = \cos^2(\tan^{-1}x)$$



Conclude $$\boxed{\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}}$$

Also (not to memorize)

$$\frac{d}{dx}(\csc^{-1}x) = \frac{-1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$$

Ex Find y'

1 $y = (\tan^{-1}x)^2$

$$y' = 2(\tan^{-1}x)\cdot \frac{1}{1+x^2}$$

2 $y = \cos^{-1}(\sin^{-1}x)$

$$y' = \frac{-1}{\sqrt{1-(\sin^{-1}x)^2}} \cdot \frac{1}{\sqrt{1-x^2}}$$

3 $y = \tan^{-1}(x^2)$

$$y' = \frac{1}{1+x^4} \cdot 2x$$

4. Show that the sum of x & y intercepts of any tangent line to $\sqrt{x}+\sqrt{y} = \sqrt{c}$ is c

A $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}y' = 0$

$$y' = \frac{-\sqrt{y}}{\sqrt{x}}$$

point $(a, (\sqrt{c}-\sqrt{a})^2)$

slope $\frac{\sqrt{a}-\sqrt{c}}{\sqrt{a}}$

$$y-(\sqrt{c}-\sqrt{a})^2 = \frac{\sqrt{a}-\sqrt{c}}{\sqrt{a}}(x-a)$$

Plug in $x=0$ & $y=0$.

Derivatives of Logs

$y = \log_b x$

$b^y = x$ apply $\left(\frac{d}{dx}\right)$

$b^y \ln b \, y' = 1$

$y' = \frac{1}{b^y \ln b} = \frac{1}{x \ln b}$

$$\boxed{\begin{aligned} \frac{d}{dx} \log_b x &= \frac{1}{x \ln b} \\ \frac{d}{dx} (\ln x) &= \frac{1}{x} \end{aligned}}$$

Ex $y = \ln(x^2 + 3)$

Ex $y = \log_2 (\cos x)$