Lecture 17

Recall $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$

Recall $y = b^x$ we calculated $\frac{dy}{dx} = \left(\lim_{h\to 0} \frac{b^h - 1}{h}\right) b^x$

what is this constant?

Answer $y = b^x = (e^{\ln b})^x = e^{\ln b\, x}$

$y' = e^{\ln b\, x} \cdot \ln b = b^x \ln b$

Conclude: $\boxed{\frac{d}{dx}(b^x) = b^x \ln b}$

so $\lim_{h\to 0} \frac{b^h - 1}{h} = \ln b$

Ex

$y = 2^{\cos x}$ $\quad \frac{dy}{dx} = (2^{\cos x} \cdot \ln 2)(-\sin x)$

$y = 3^{x^2}$ $\quad y' = (3^{x^2} \ln 3)(2x)$

$y = x^x$ $\quad$ * Not of form x^n or b^x

we cannot find $\frac{dy}{dx}$ yet.

Summary of differentiation rules Assume $u = g(x)$ is differentiable

$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$

$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$, $\frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$, $\frac{d}{dx}\tan u = \sec^2 u \frac{du}{dx}$
+ 3 more

$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$

+ sum, product, quotient rules

$\frac{d}{dx}(b^u) = b^u \ln b \frac{du}{dx}$

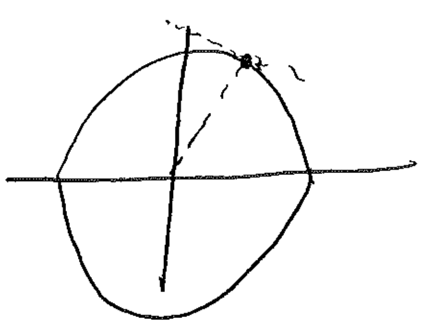
Ex $y = (x^3+2x+1)^7(x^5-2x+6)^3$

$y' = 7(x^3+2x+1)^6(3x^2+2)(x^5-2x+6)^3 + (x^3+2x+1)^7 \cdot 3(x^5-2x+6)^2(5x^4-2)$

Implicit Differentiation

Problem Given a curve $f(x,y)=0$, can we find $\frac{dy}{dx}$? tang lines

Ex $x^2+y^2=25$



Find tang line at $(3,4)$

Method 1: $y = \sqrt{25-x^2}$ is top of circle

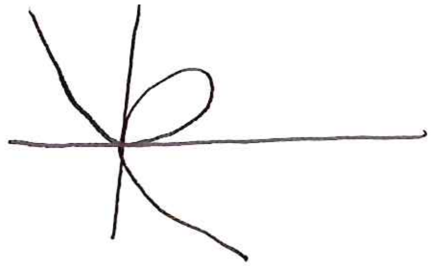
$y' = \frac{1}{2\sqrt{25-x^2}} \cdot (-2x) = \frac{-2x}{2\sqrt{25-x^2}}$

$y'(3) = \frac{-6}{2\sqrt{16}} = -3/4$

$\boxed{y-4 = -\frac{3}{4}(x-3)}$

Problem

$x^3+y^3=6xy$



Find tang line
at point

Cannot easily solve for y in terms of x.

Implicit differentiation

Assume y is implicitly a function of x.

Apply $\frac{d}{dx}$.

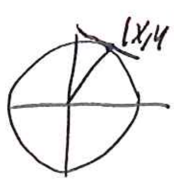
So $\left(\frac{d}{dx}\right)(y)=\frac{dy}{dx}$

$\left(\frac{d}{dx}\right)(x)=1$

Example $x^2+y^2=25$. Apply $\frac{dy}{dx}$:

$$\underbrace{2x+2y\frac{dy}{dx}}_{\text{from Chain Rule}}=0. \Rightarrow \frac{dy}{dx}=-\frac{x}{y}.$$

Remarks

1. 

This proves tangent line is $\perp$ to radius

2. When $y=0$ $\frac{dy}{dx}$ does not exist (vertical tangent)

Problem Find tang line to $x^3+y^3=6xy$ at $(3,3)$

Solution $3x^2+3y^2\frac{dy}{dx}=6y+6x\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{3x^2-6y}{6x-3y^2}$

at $(3,3)$ $\frac{dy}{dx}=\frac{27-18}{18-27}=-1$

$$\boxed{y-3=-1(x-3)}$$

Problem Find hor. tangents

Problem

$e^y\sin x = x+xy$. Find y'

Ex $x^2+y^2=(2x^2+2y^2-x)^2$. Find tangent line at $(0,\frac{1}{2})$

Ex $x^3-y^3=7$ Find y'' by implicit diff.