SOLUTIONS

Math 141- Midterm Exam #2 - October 24, 2016

1. (50 points) Find $\frac{dy}{dx}$. You do not need to simplify your answers.

a. $y = \frac{\tan x}{x^3+x+1}$

$$y' = \frac{(x^3+x+1)\sec^2 x - \tan x\,(3x^2+1)}{(x^3+x+1)^2}$$

b. $y = xe^{3x}$

$$y' = e^{3x} + 3xe^{3x}$$

c. $y = 3^{\sec x}$

$$y' = 3^{\sec x} \ln 3 \cdot \sec x \tan x$$

d. $y = (6 + x^3 + 2x^5)^{10}$

$$y' = 10(6+x^3+2x^5)^9(3x^2+10x^4)$$

e. $xy + e^y = 5$

$y + xy' + e^y y' = 0$

$y' = \dfrac{-y}{x+e^y}$

f. $y = \tan^{-1} x$

$y' = \dfrac{1}{1+x^2}$

g. $y = (\ln(x^2 + \sin x))^5$

$y' = 5(\ln(x^2+\sin x))^4 \cdot \dfrac{1}{x^2+\sin x} \cdot (2x+\cos x)$

h. $y = \log_5 x$

$$y' = \frac{1}{x \ln 5}$$

i. $y = x^{\tan x}$

$$\ln y = \tan x \ln x$$

$$\frac{1}{y} y' = \sec^2 x \ln x + \frac{\tan x}{x}$$

$$\boxed{y' = x^{\tan x}\left(\sec^2 x \ln x + \frac{\tan x}{x}\right)}$$

j. $y = \frac{(x^2+5)^{10}(\cos x)(x^3+1)^3}{x^2+3}$

$$\ln y = 10\ln(x^2+5) + \ln(\cos x) + 3\ln(x^3+1) - \ln(x^2+3)$$

$$\frac{1}{y} y' = \frac{20x}{x^2+5} - \frac{\sin x}{\cos x} + \frac{9x^2}{x^3+1} - \frac{2x}{x^2+3}$$

$$y' = y \cdot \left(\quad '' \quad \right)$$

2. **(10 points)** A sample of tritium-3 decayed to 94.5% of its original amount in one year. What is its half life?

$m(t) = m(0)\, e^{kt}$

$.945\, m(0) = m(0)\, e^{k \cdot 1}$

$.945 = e^{k}$

$k = \ln(.945)$

$\text{Half-life} = \dfrac{-\ln 2}{k} = \dfrac{-\ln 2}{\ln(.945)}$

3. **(5 points)** If $F(x) = f(g(x))$ where $f(-4) = 2, f'(-4) = 5, f'(4) = 3, g(4) = -4$, and $g'(4) = 8$, find $F'(4)$.

$F'(4) = f'(g(4)) \cdot g'(4)$

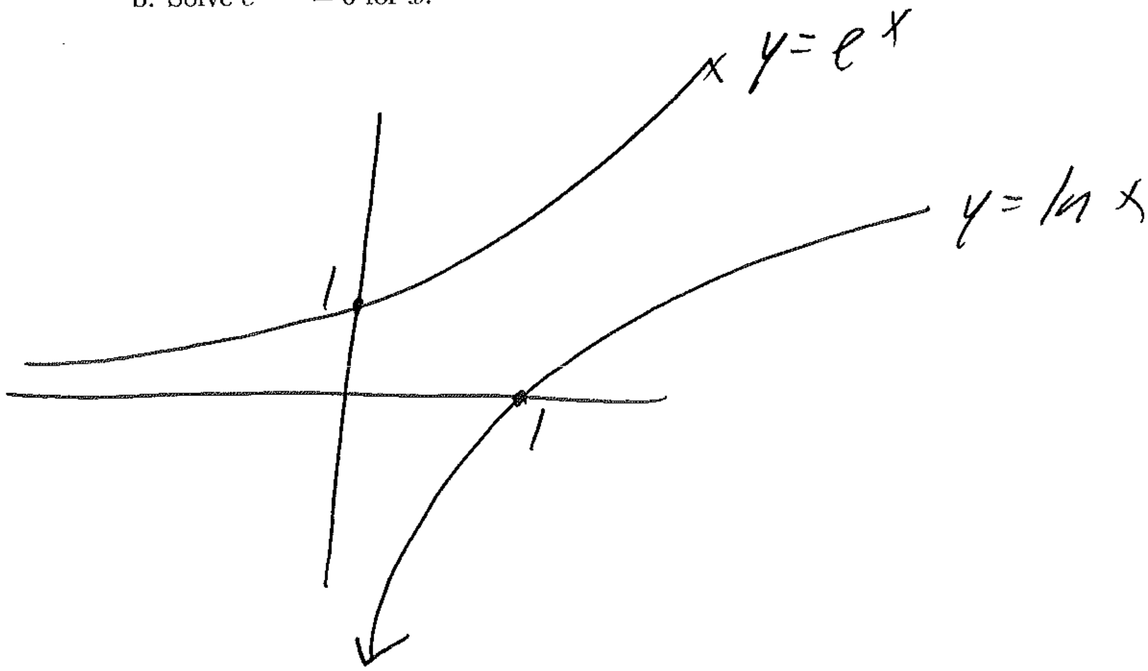
$= f'(-4) \cdot 8$

$= 5 \cdot 8$

$= 40$

4. **(10 points)**

a. Sketch the graph of $y = e^x$ and $y = \ln x$ on the same axes, labelling any intercepts.
b. Solve $e^{7-4x} = 6$ for x.



b $7 - 4x = \ln 6$

$$x = \frac{7 - \ln 6}{4}$$

5. **(10 points)** Find the equation of the tangent line to the curve $x^2y^3 - x^3y^2 = 12$ at $(-1, 2)$.

$$2xy^3 + x^2 \cdot 3y^2y' - 3x^2y^2 - x^3 \cdot 2yy' = 0$$

$$-16 + 12y' - 12 + 4y' = 0$$

$$16y' = 28$$

$$y' = 7/4$$

$$y - 2 = 7/4\,(x+1)$$

6. **(10 points)** Find the linearization of the function $f(x) = \sqrt{x+3}$ and $a = 1$. Use it to approximate $\sqrt{4.05}$. Is your approximation and underestimate or overestimate?

$$f'(x) = \frac{1}{2\sqrt{x+3}}$$

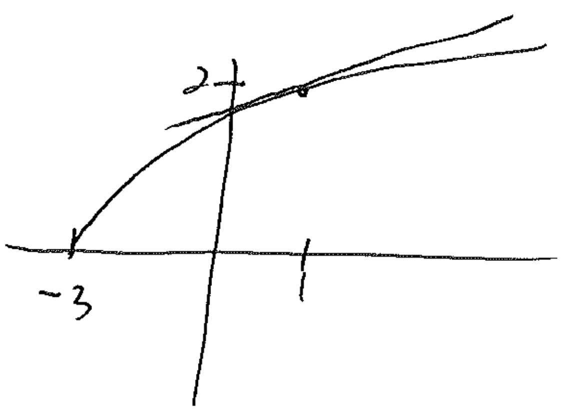
$$f(a) = 2 \qquad f'(a) = 1/4$$

$$L(x) = f(a) + f'(a)(x-a)$$

$$\boxed{L(x) = 2 + \frac{1}{4}(x-1)}$$

$\sqrt{4.05}$ is $f(1.05)$

$$L(1.05) = 2 + \frac{1}{4}(1.05-1) = 2 + \frac{.05}{4}$$



overestimate, tangent line is above graph

Name:

7. **(5 points)** Recall that the volume of a circular cylinder of length h and radius r is $V = \pi r^2 h$. Suppose a lump of modeling clay is being rolled out so that it maintains always the shape of a circular cylinder. Suppose the length h is increasing a a rate proportional to itself (i.e. $\frac{dh}{dt} = \lambda h$ for some constant λ). Prove that the radius is decreasing at a rate proportional to itself. Hint: The volume of clay remains constant.

Given $\frac{dV}{dt} = 0$, $\frac{dh}{dt} = \lambda h$

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$$

$$0 = 2\pi r h \frac{dr}{dt} + \pi r^2 \lambda h$$

$$0 = \pi r h \left(2 \frac{dr}{dt} + \lambda r\right)$$ Now $\pi r h \neq 0$

so $$0 = 2 \frac{dr}{dt} + \lambda r$$

$$\boxed{\frac{dr}{dt} = \frac{-\lambda}{2} r}$$ as desired