Name: SOLUTIONS

Math 141B- Midterm Exam #3 - November 18, 2016

1. **(10 points)**

a. Let $f(x)$ be a function with domain $D$. Say $f$ has an absolute (or global) maximum value on $D$ at the point $x = c$ if ...

$f(c) \geq f(x)$ for all $x \in D$.

b. A point $(c, f(c))$ on the graph of $y = f(x)$ is called an *inflection point* if ....

the concavity changes at that point.

2. **(5 points)** State the Mean Value Theorem, including any necessary hypotheses.

Suppose $f(x)$ is continuous on $[a,b]$ and differentiable on $(a,b)$.

Then there exists $c \in (a,b)$ with

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

3. **(15 points)** Let $f(x) = x^3 - 3x^2 - 9x + 1$. Find the global maximum and minimum values of $f(x)$ on the interval $[-2, 10]$.

$f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3)$
$= 3(x-3)(x+1)$

crit values $x = -1$, $x = 3$

| $x$ | $f(x)$ |
|---|---|
| $-2$ | $-1$ |
| $-1$ | $6$ |
| $3$ | $-26$ |
| $10$ | $611$ |

global max value $= 611$
global min value $= -26$

$f(-2) = -8 - 12 + 18 + 1 = -1$
$f(-1) = -1 - 3 + 9 + 1 = 6$
$f(3) = 27 - 27 - 27 + 1 = -26$
$f(10) = 1000 - 300 - 90 + 1 = 611$

4. **(10 points)** Use Newton's method to estimate the one real solution of $x^3 + x + 1 = 0$. Start with an initial guess of $x_1 = 0$ and then find $x_2$ and $x_3$.

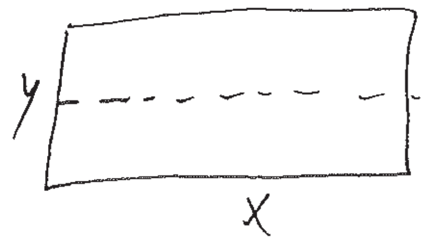
$$f'(x) = 3x^2 + 1$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_2 = 0 - \frac{1}{1} = -1$$

$$x_3 = -1 - \frac{-1}{4} = -.75$$

5. **(20 points)** A 216 square meter rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?




Given $xy = 216$. Minimize $L = 3x + 2y$

$$y = \frac{216}{x}$$

$$L(x) = 3x + \frac{432}{x}$$

$$L'(x) = 3 - \frac{432}{x^2} \qquad \text{set} = 0$$

$$3x^2 = 432$$

$$x^2 = 144$$

$$x = 12 \rightarrow y = 18$$

dimensions $12 \times 18$

Length of fence $= 3 \cdot 12 + 2 \cdot 18 = 72$ ft

6. **(10 points)** Calculate the following limit:

$$\lim_{x \to 0} \frac{8x^2}{\cos x - 1}. \quad \text{type } \frac{0}{0}$$

$$\underset{=}{\text{LHR}} \lim_{x \to 0} \frac{16x}{-\sin x} \quad \text{type } \frac{0}{0}$$

$$\underset{=}{\text{LHR}} \lim_{x \to 0} \frac{16}{-\cos x} = -16$$

7. **(20 points)** Let

$$f(x) = \frac{x^2 - 1}{x^3}.$$

Using the quotient rule we obtain:

$$f'(x) = \frac{3 - x^2}{x^4}, \quad f''(x) = \frac{2x^2 - 12}{x^5}.$$

a. Find all $x$ and $y$ intercepts and any vertical or horizontal asymptotes. Determine the behavior of the graph on either side of any vertical asymptotes.

b. Find the intervals where $f(x)$ is increasing or decreasing and any local maximums or local minimums.

c. Find the intervals where $f(x)$ is concave up or concave down, and determine any inflection points.

d. Neatly sketch the graph of $y = f(x)$, Label the $x$ and $y$ coordinates of any intercepts, local extrema and inflection points.

a. intercepts $(1,0)$, $(-1,0)$

H.A $y = 0$

V.A. $x = 0$

$\lim_{x \to 0^+} f(x) = -\infty$ $\quad$ $\lim_{x \to 0^-} f(x) = \infty$

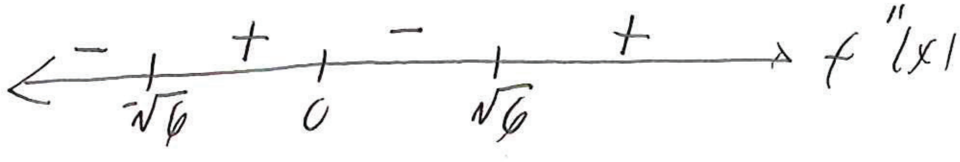
b

$$\begin{array}{ccccccccc} & - & & + & & + & & - & \\ \longleftarrow & & | & & | & & | & & \longrightarrow \; f' \\ & & -\sqrt{3} & & 0 & & \sqrt{3} & & \end{array}$$

decreasing $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$ $\quad$ increasing $(-\sqrt{3}, 0) \cup (0, \sqrt{3})$

local min $\left(-\sqrt{3}, \frac{2}{-3\sqrt{3}}\right)$ $\quad$ local max $\left(\sqrt{3}, \frac{2}{3\sqrt{3}}\right)$

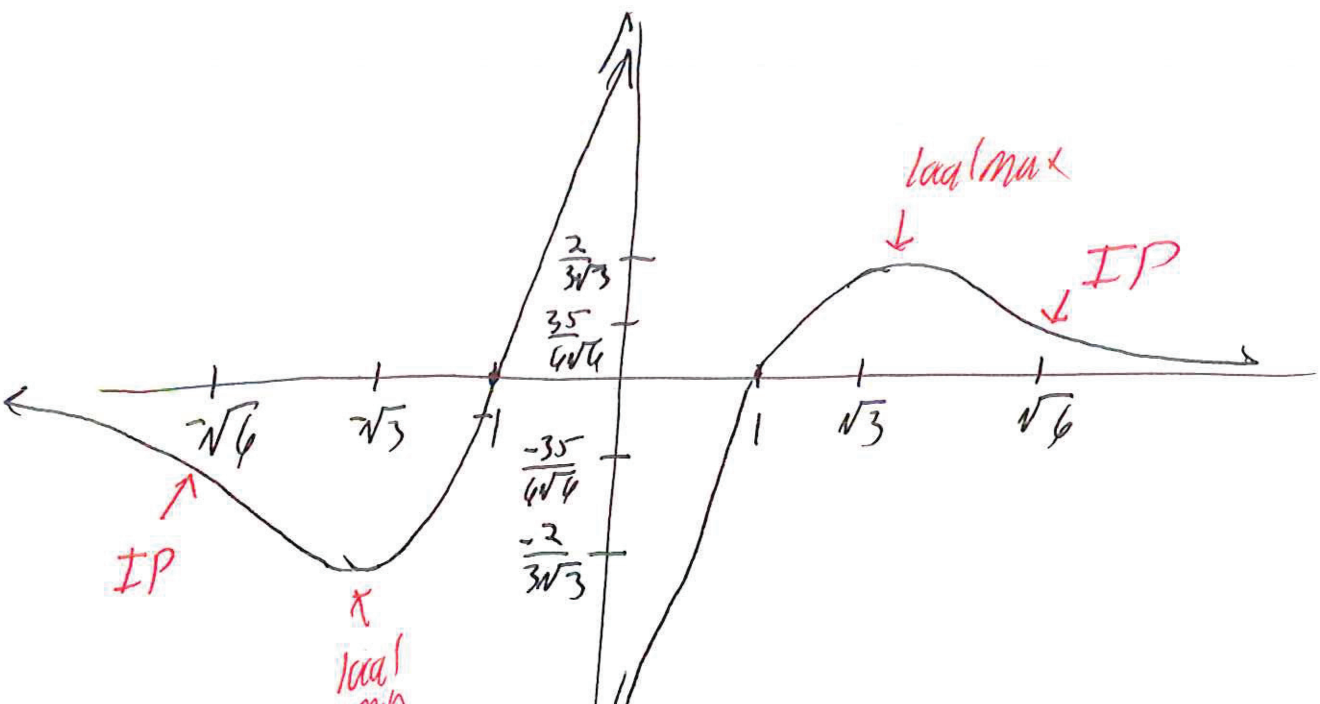
c.




concave down $(-\infty, -\sqrt{6}) \cup (0, \sqrt{6})$

concave up $(-\sqrt{6}, 0) \cup (\sqrt{6}, \infty)$

I.P. $\left(-\sqrt{6}, \frac{-35}{6\sqrt{6}}\right)$ $\left(\sqrt{6}, \frac{35}{6\sqrt{6}}\right)$



# Name:

8. **(10 points)** Suppose $f'(x) = (x-2)^2(x+3)(x-3)$. (Pay attention, you are given $f'(x)$ not $f(x)$!)
a. Find the critical values.
b. Find the intervals on which $f(x)$ is increasing or decreasing.
b. Classify each critical value as a local max, min or neither using the first derivative test.

a. $X = 2, -3, 3$

b. $f'$: $+$ on the left of $-3$, $-$ between $-3$ and $2$, $-$ between $2$ and $3$, $+$ on the right of $3$

$$\xleftarrow{\quad + \quad} -3 \xrightarrow{\quad - \quad} 2 \xrightarrow{\quad - \quad} 3 \xrightarrow{\quad + \quad} \quad f'$$

$f'(4) = 4 \cdot 7 \cdot 1 = 28 > 0$

$f'(2.5) = \frac{1}{4}(5.5)(-.5) < 0$

$f'(0) = 4(3)(-3) < 0$

$f'(-4) = 36 \cdot (-1)(-7) > 0$

increasing $(-\infty, -3) \cup (3, \infty)$

decreasing $(-3, 3)$

c. local max at $X = -3$

local min at $X = 3$

Neither at $X = 2$