SOLUTIONS

Math 141B- Midterm Exam #1 - September 26, 2016

1. **(15 points)** True or false:

F a. The graph of a function can have infinitely many horizontal asymptotes.

T b. If $f'(a)$ exists than $\lim_{x\to a} f(x) = f(a)$.

T c. The graph of a cubic polynomial $y = ax^3 + bx^2 + cx + d$ has at the most two horizontal tangent lines.

F d. $f(x) = \sqrt[3]{x}$ is differentiable at $x = 0$.

F e. $f(x) = \tan x$ is continuous on $(-\infty, \infty)$.

2. **(5 points)** State the Intermediate Value Theorem.

Let $f(x)$ be continuous on $[a,b]$.
Suppose N is between $f(a)$ and $f(b)$.
Then there is a c in (a,b) with $f(c)=N$.

3. **(20 points)** Evaluate the following limits. If the limit does not exist then write DNE.

a.

$$\lim_{x\to-\infty} \frac{\sqrt{1+6x^6}}{2-x^3} = \lim_{x\to-\infty} \frac{\frac{\sqrt{1+6x^6}}{x^3}}{\frac{2}{x^3}-1}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{\frac{1}{x^6}+6}}{\frac{2}{x^3}-1}$$

$$= \sqrt{6}$$

b.

$$\lim_{x\to3^-} \frac{6-x}{(x-3)(x+2)}$$

$x = 2.99$

3.01

f. 01 | 5 |

$-\infty$

c.

$$\lim_{x\to6} \sin x.$$

sin 6

d. Suppose $\lim_{x\to2} f(x) = 4$ and $\lim_{x\to2} g(x) = -1$. Evaluate $\lim_{x\to2} \frac{2f(x)+g(x)^2}{\sqrt{f(x)}}$.

$$\frac{2\cdot4+(-1)^2}{\sqrt{4}} = 9/2$$

4. **(10 points)** Find the formula for a single function $f(x)$ that satisfies the following four conditions:

- $$\lim_{x\to\pm\infty} f(x) = 0$$
- $$\lim_{x\to 0} f(x) = -\infty.$$
- $$\lim_{x\to 5^-} f(x) = \infty, \lim_{x\to 5^+} f(x) = -\infty$$
- $$f(2) = 0.$$

$$-\frac{(x-2)}{x^2(x-5)}$$

5. **(20 points)** Let $f(t) = \frac{1}{2t+1}$.

a. *From the definition* of the derivative, calculate $f'(t)$.

b. Using your answer from part a, determine the equation of the tangent line to the graph when $t = 1$.

$$f'(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h} = \lim_{h \to 0} \frac{\frac{1}{2t+2h+1} - \frac{1}{2t+1}}{h}$$

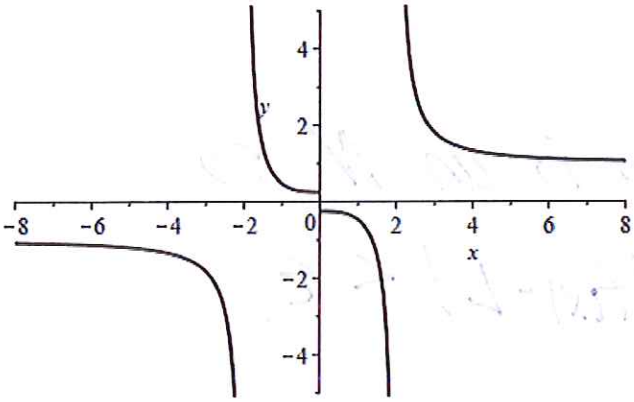
$$= \lim_{h \to 0} \frac{\frac{2t+1-(2t+2h+1)}{(2t+1)(2t+2h+1)}}{h} = \lim_{h \to 0} \frac{-2h}{h(2t+1)(2t+2h+1)}$$

$$= \frac{-2}{(2t+1)^2}$$

$$\boxed{f'(t) = \frac{-2}{(2t+1)^2}}$$

b. $f(1) = 1/3$ $\quad f'(1) = \frac{-2}{9}$

$$\boxed{y - 1/3 = -2/9(x-1)}$$



6. **(15 points)** Above is the graph of a function $y = f(x)$.

a. Find the vertical and horizontal asympotes.

V.A. $X = -2,\ X = 2$ HA $y = 1,\ y = -1$

b. Estimate $\lim_{x \to 0^-} f(x)$.

$1/4$

c. Estimate $f'(-1)$.

-1

d. Is $\boxed{f''(4) > 0}$ or < 0? Explain.

. f' is negative but getting less negative so f'' is positive.

e. List the x values where $f(x)$ is discontinuous and, for each, state which type of discontinuity it is.

infinite disc. at $X = \pm 2$

jump disc. at $X = 0$

Name:

7. **(15 points)**
 a. Give the precise definition for $\lim_{x\to\infty} f(x) = L$.

For any $\varepsilon > 0$ there is an N so if $x > N$ then $|f(x) - L| < \varepsilon$.

b. Use the definition to prove that

$$\lim_{x\to\infty} \frac{1}{3x} = 0.$$

Let $\varepsilon > 0$ be given. Choose $N = 1/3\varepsilon$.

Suppose $x > N$. Then $x > \frac{1}{3\varepsilon}$ so

$3\varepsilon x > 1$ so

$\varepsilon > \frac{1}{3x}$ since $x > 0$.

Thus $|f(x) - 0| = \left|\frac{1}{3x}\right| < \varepsilon$ as desired.