SOLUTIONS

Math 141A- Midterm Exam #1 - September 25, 2014

1. **(15 points)** True or false:

_T_ a. A function which is differentiable at $x = a$ must also be continuous at $x = a$.

_T_ b. The composition of two continuous functions is also continuous.

_F_ c. Suppose $f(x)$ is continuous on $[0, 2]$ and $f(0) = 2, f(2) = 5$. Then the intermediate value theorem implies $f(x)$ does not have a root in $(0, 2)$.

_F_ d. Suppose $y = a$ is a horizontal asymptote for $f(x)$. Then the graph of $y = f(x)$ does not cross the line $y = a$.

_T_ e. $f(x) = \frac{\sin x}{x}$ has a removable discontinuity at $x = 0$.

2. **(20 points)**
a. Give the $\epsilon - \delta$ definition for $\lim_{x \to a} f(x) = L$.

For any $\varepsilon > 0$ there is a δ such that if $0 < |x-a| < \delta$ then $|f(x) - L| < \varepsilon$.

b. Use the definition to prove that

$$\lim_{x \to 3} (2x + 8) = 14.$$

Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/2$. Suppose $0 < |x-3| < \delta$. We must prove $|f(x) - L| < \varepsilon$ where $f(x) = 2x+8$ and $L = 14$. But

$$|f(x) - L| = |2x+8-14| = |2x-6| = 2|x-3| < 2\delta = \varepsilon.$$

So $|f(x) - L| < \varepsilon$ as desired. //

3. **(20 points)** Evaluate the following limits. If the limit does not exist then write DNE.

a. $\lim_{x\to -\infty} \frac{\sqrt{x^2+5}}{2x-3}$

Since $x<0$ $x = -\sqrt{x^2}$. Divide top & bottom by x

$$\lim_{x \to -\infty} \frac{-\sqrt{1+\frac{5}{x^2}}}{2-\frac{3}{x}} = -1/2$$

b. $\lim_{x\to 3^+} \frac{5-3x}{(x-3)(x-5)}$

Try $x = 3.001$

$$\approx \frac{-4}{(.001)(-2)}$$

∞

c. $\lim_{x\to 6} \sin x$.

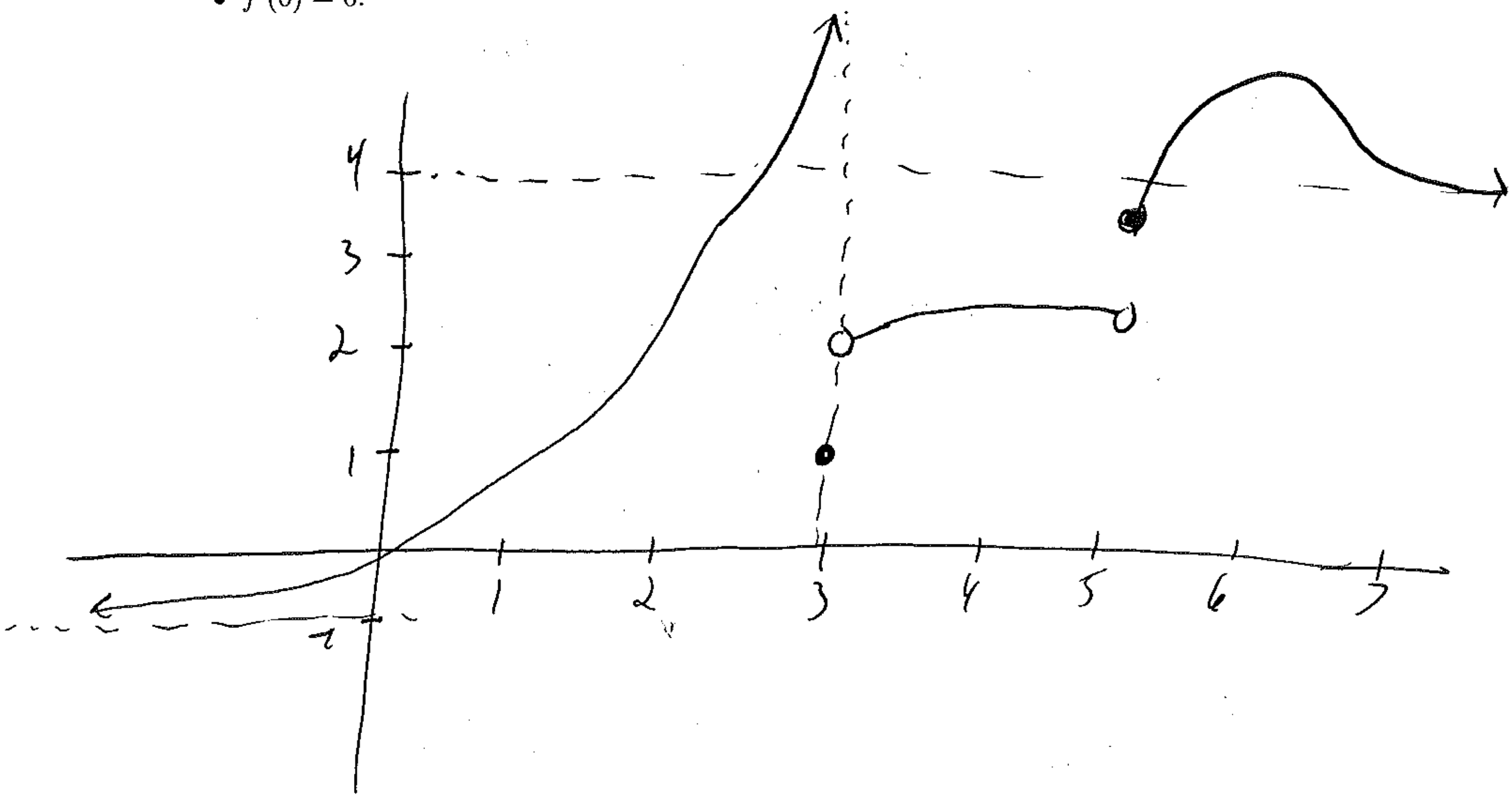
$\sin 6$

d. Suppose $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 2} g(x) = -1$. Evaluate $\lim_{x\to 2} \frac{f(x)+3(g(x)^2)}{\sqrt{f(x)}}$.

$6/\sqrt{3}$

4. **(10 points)** Neatly sketch the graph of a single function $f(x)$ which has the following properties:

- $\lim_{x\to 3^+} f(x) = 2$, $\lim_{x\to 3^-} f(x) = \infty$, $f(3) = 1$.
- $f(x)$ is continuous from the right at $x = 5$ but not continuous from the left at $x = 5$.
- $\lim_{x\to\infty} f(x) = 4$, $\lim_{x\to-\infty} f(x) = -1$.
- $f'(6) = 0$.



5. **(20 points)** A ball is tossed and has height in feet given at time t seconds by $y(t) = -t^2 + 6t$.

a. *Use the definition* of the derivative to prove that $y'(t) = -2t + 6$.

$$y'(t) = \lim_{h\to 0} \frac{y(t+h)-y(t)}{h} = \lim_{h\to 0} \frac{-(t+h)^2+6(t+h)-(-t^2+6t)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{-t^2}-2th-h^2\cancel{+6t}+6h\cancel{+t^2}\cancel{-6t}}{h}$$

$$= \lim_{h\to 0} \frac{-2th+6h-h^2}{h} = \lim_{h\to 0} -2t+6-h$$

$$= -2t+6$$

b. Find the equation of the tangent line to $y = y(t)$ at the point where $t = 2$.

point $(2,8)$

slope $= y'(2) = 2$

$$\boxed{y-8=2(t-2)}$$

c. What is the ball's average velocity from time $t = 1$ to time $t = 3$?

$$\frac{y(3)-y(1)}{3-1} = \frac{9-5}{3-1} = 2 \text{ ft/sec}$$

d. How fast is the ball moving when it hits the ground?

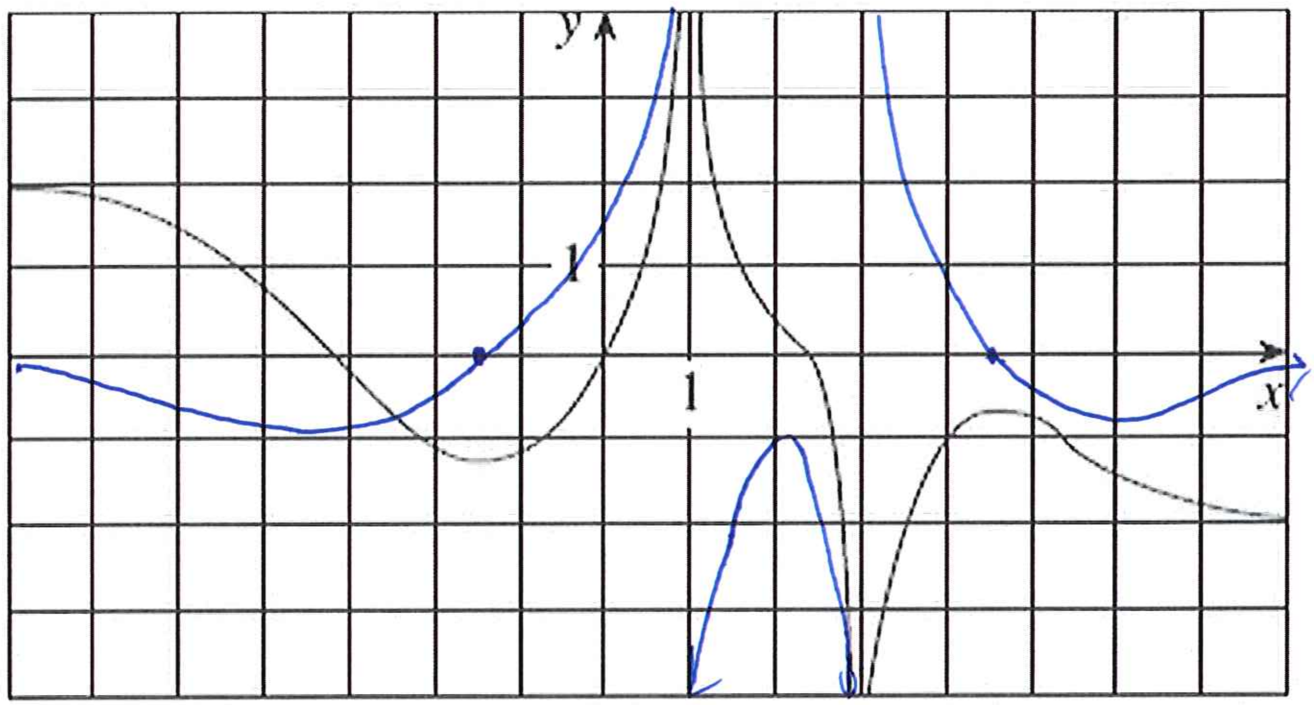
$-t^2+6t=0 \quad t=0,6$

$y'(6) = -12+6 = -6$

$\boxed{\text{going } 6 \text{ ft/sec down}}$

e. What is the ball's acceleration?

accell $= y'' = -2 \text{ ft/sec}^2$



6. **(15 points)** Above is the graph of a function $y = f(x)$.

a. Find the vertical and horizontal asymptotes.

$\boxed{\begin{array}{lll} x=1 & x=3 & V.A. \\ y=2 & y=-2 & H.A. \end{array}}$

b. Evaluate $\lim_{x\to\infty} f(x)$.

-2

c. Estimate $f'(0)$.

2

d. Estimate $\lim_{x\to\infty} f'(x)$.

0

e. On the same axes above carefully sketch a graph of $y = f'(x)$

Name: