Name: SOLUTIONS

Math 141- Midterm Exam #2 - October 24, 2007

1. **(50 points)** Find $\frac{dy}{dx}$. You do not need to simplify your answers.

a. $y = x\cos(x)$

$$y' = \cos x - x\sin x$$

b. $y = \tan(x^2+1)$

$$y' = 2x\sec^2(x^2+1)$$

c. $y = \frac{\sqrt{x}}{x^3+1}$

$$y' = \frac{(x^3+1)\left(\frac{1}{2\sqrt{x}}\right) - \sqrt{x}\cdot 3x^3}{(x^3+1)^2}$$

d. $y = \ln(5x-15)$

$$y' = \frac{5}{5x-15}$$

e. $y = 3^x$

$$y' = 3^x \ln 3$$

f. $y = x^{10} - 9x^3 + 15x - 3$

$$y' = 10x^9 - 27x^2 + 15$$

g. $y = \sin^3(x^2)$

$$y' = 3\sin^2(x^2)\cos(x^2)\cdot 2x$$

h. $y = (\sin x)^x$.

$$\ln y = x\ln\sin x$$

$$\frac{1}{y}y' = \ln\sin x + \frac{x\cdot\cos x}{\sin x} \qquad y' = (\sin x)^x\cdot\left(\ln\sin x + x\cot x\right)$$

i. $y = \sqrt{\frac{(x^2+1)^5 e^x x^9}{x^2+2}}$

$$\ln y = \frac{1}{2}\left(5\ln(x^2+1) + x\ln e + 9\ln x - \ln(x^2+2)\right)$$

$$\frac{1}{y}y' = \frac{1}{2}\left(\frac{10x}{x^2+1} + 1 + \frac{9}{x} - \frac{2x}{x^2+2}\right)$$

$$y' = \frac{1}{2}\sqrt{\frac{(x^2+1)^5 e^x x^9}{x^2+2}}\left(\frac{10x}{x^2+1} + 1 + \frac{9}{x} - \frac{2x}{x^2+2}\right)$$

j. $\sin(xy) = 5x$

$$\cos(xy)(y + xy') = 5$$

$$y\cos(xy) + y'x\cos(xy) = 5$$

$$y' = \frac{5 - y\cos(xy)}{x\cos(xy)}$$

2. **(10 points)** Find the equation of the tangent line to the curve

$$2x - xy^2 = -6$$

at the point $(3, 2)$.

$2 - y^2 - x2yy' = 0$

$2 - 4 - 12y' = 0$

$y' = -2/12 = -1/6$

$y - 2 = -\frac{1}{6}(x-3)$

3. **(10 points)** Find the linear approximation to the function $f(x) = x^{3/4}$ at x=16. Then use this linear approximation to estimate $15^{3/4}$.

$f(x) = x^{3/4}$ $\quad f'(x) = \frac{3}{4}x^{-1/4} = \frac{3}{4\sqrt[4]{x}}$

$f(16) = 16^{3/4} = 8$ $\quad f'(16) = \frac{3}{4 \cdot 2} = 3/8$

$\boxed{L(x) = \frac{3}{8}(x-16) + 8}$

$15^{3/4} = f(15) \approx L(15) = \frac{3}{8}(15-16) + 8$

$= -3/8 + 8$

$= 61/8$

4. (**5 points**) Evaluate this limit by *first expressing it as a derivative*:

$$\lim_{h \to 0} \frac{\sqrt[4]{16+h} - 2}{h}.$$

If $f(x) = x^{1/4}$ then this limit is $f'(16)$

$$f'(x) = \frac{1}{4} x^{-3/4} = \frac{1}{4(\sqrt[4]{x})^3}$$

$$f'(16) = \frac{1}{32}$$

5. (**10 points**) Let $r(x) = f(g(h(x)))$, where $h(1) = 2$, $h'(2) = -1$, $g(2) = 3$, $h'(1) = 4$, $g'(2) = 5$, and $f'(3) = 6$. Find $r'(1)$.

$$r'(x) = f'(g(h(1))) \cdot g'(h(1)) \cdot h'(1)$$

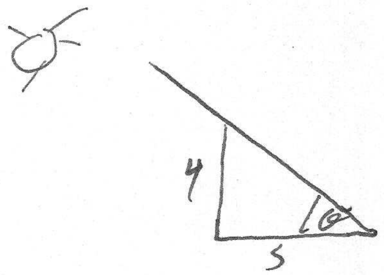
$$= f'(g(2)) \cdot g'(2) \cdot 4$$

$$= f'(3) \cdot 5 \cdot 4$$

$$= 6 \cdot 5 \cdot 4$$

$$= 120$$

6. (**15 points**) The angle of elevation of the sun is decreasing at a rate of 0.25 radians/hour. How fast is the length of the shadow cast by a 4 foot tall pole increasing when the angle of elevation of the sun is $\pi/6$? (FYI: $\cos(\pi/6) = \sqrt{3}/2$, $\sin(\pi/6) = 1/2$.)



Given: $\frac{d\theta}{dt} = -.25$

Find $\frac{ds}{dt}$ when $\theta = \pi/6$

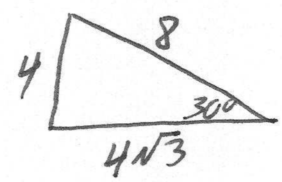
$\tan\theta = \frac{4}{s}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{-4}{s^2} \frac{ds}{dt}$

When $\theta = \pi/6$:

$\tan 30 = 1/\sqrt{3}$

$\frac{4}{s} = \frac{1}{\sqrt{3}} \rightarrow s = 4\sqrt{3}$



$\sec^2(\pi/6) = \frac{1}{\cos^2(\pi/6)} = \frac{1}{3/4} = 4/3$

$\frac{4}{3}\left(-\frac{1}{4}\right) = \frac{-4}{48} \cdot \frac{ds}{dt}$

$\frac{1}{3} = \frac{1}{12} \frac{ds}{dt}$

$\frac{ds}{dt} = 4$ feet/hour