Name: SOLUTIONS

Math 141- Midterm Exam #1 - September 24, 2007

1. **(15 points)** True or false:

F a. A function which is continuous at $x = a$ must also be differentiable at $x = a$.

T b. It is possible for the graph of a function to have 3 vertical asymptotes.

F c. The intermediate value theorem applies to $f(x) = 1/x$ on the interval $[-2, 1]$.

F d. If $\lim_{x \to 0} f(x) = \infty$ and $\lim_{x \to 0} g(x) = \infty$ then $\lim_{x \to 0}[f(x) - g(x)] = 0$

T e. If $p(x)$ is a polynomial then $\lim_{x \to 5} p(x) = p(5)$.

2. **(20 points)**

a. Give the formal definition for $\lim_{x \to a} f(x) = L$.

For any $\varepsilon > 0$ there is a $\delta > 0$ such that

if $0 < |x-a| < \delta$ then $|f(x) - L| < \varepsilon$.

b. Use the definition to prove that

$$\lim_{x \to 4} (3x - 7) = 5.$$

Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/3$

Suppose $0 < |x-4| < \delta$. Then

$|f(x) - L| = |3x - 7 - 5| = |3x - 12|$

$= 3|x-4| < 3\delta = \varepsilon$.

Thus $|f(x) - L| < \varepsilon$ as required. //

3. (**20 points**) Evaluate the following limits. If the limit does not exist then write DNE.

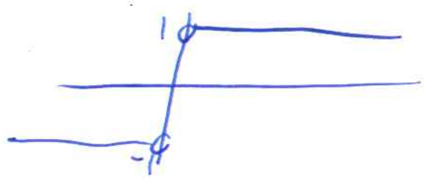
a. $\lim_{x\to -3} \frac{x^2-9}{x^2+2x-3}$

$$= \lim_{x\to -3} \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x-1)}$$

$$= \lim_{x\to -3} \frac{x-3}{x-1} = \frac{-6}{-4} = 3/2$$

b. $\lim_{x\to 0} \frac{|x|}{x}$

DNE



c. $\lim_{x\to -\infty} \frac{\sqrt{x^2-9}}{2x-6}$. For $x<0$, $x=-\sqrt{x^2}$

$$= \lim_{x\to -\infty} \frac{\frac{\sqrt{x^2-9}}{x}}{2-6/x} = \lim_{x\to -\infty} \frac{-\sqrt{1-9/x^2}}{2-6/x}$$

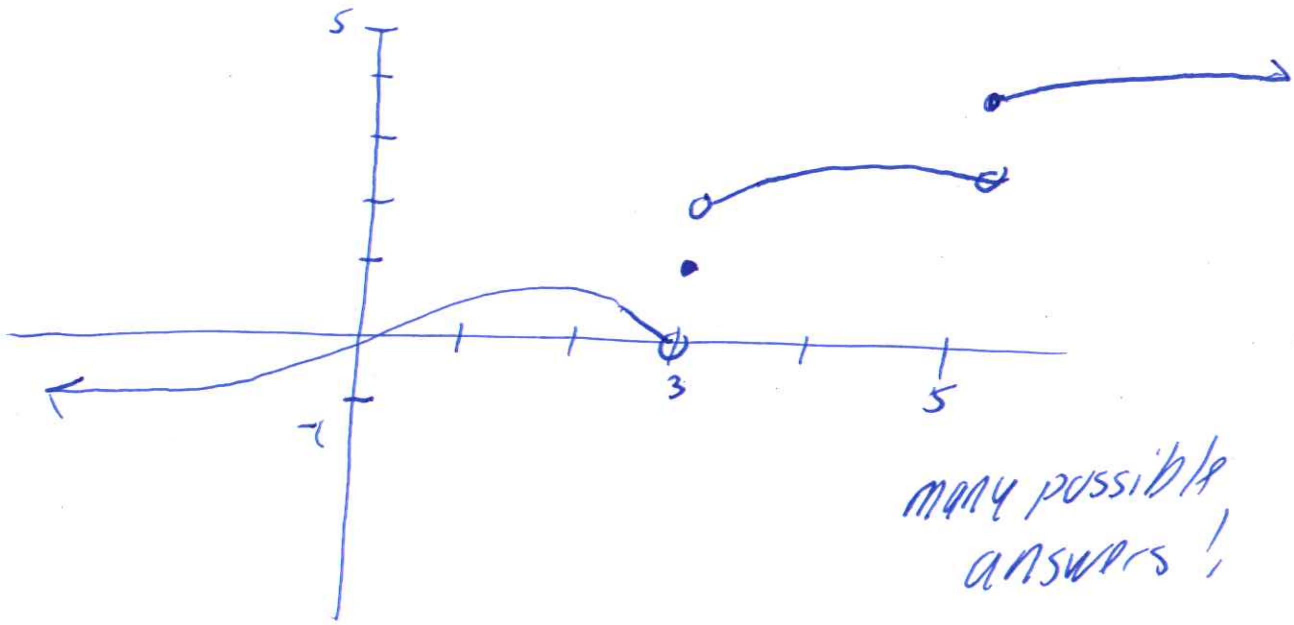
$$= -1/2$$

d. $\lim_{x\to\infty} \frac{2x^2-5x+11}{x^2-2}$.

$= 2$

4. **(15 points)** a. Neatly sketch the graph of a single function $f(x)$ which has the following properties:

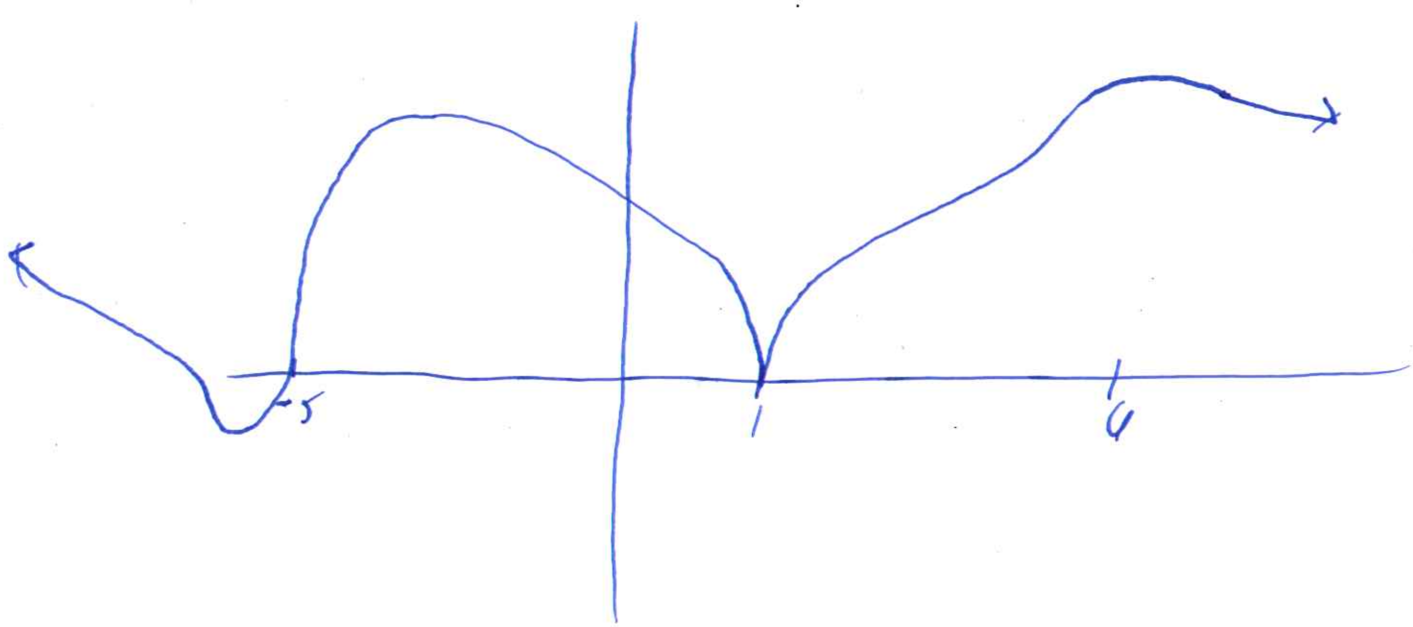
- $\lim_{x\to 3^+} f(x) = 2$, $\lim_{x\to 3^-} f(x) = 0$, $f(3) = 1$.
- $f(x)$ is continuous from the right at $x = 5$ but not continuous from the left at $x = 5$.
- $\lim_{x\to\infty} f(x) = 4$, $\lim_{x\to-\infty} f(x) = -1$.



many possible answers!

b. Neatly sketch the graph of a single function $g(x)$ which has the following properties:

- $g(x)$ is continuous on $(-\infty, \infty)$
- $g'(6) = 0$
- $g(x)$ is not differentiable at $x = 1$
- $g(x)$ has a vertical tangent line at $x = -5$.



5. **(20 points)** Let $f(x) = 1/x$.

a. *Use the definition* of the derivative to prove that $f'(x) = -1/x^2$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{-h}{hx(x+h)} = \lim_{h\to 0} \frac{-1}{x(x+h)}$$

$$= -1/x^2$$

b. Find the equation of the tangent line to $y = 1/x$ at the point where $x = 5$.

slope $= -1/25 = f'(5)$ point $= (5, 1/5)$

$$\boxed{y - 1/5 = -1/25(x-5)}$$

6. **(10 points)** The graph of a function $f(x)$ is given below. Use it to sketch the graph of the derivative $f'(x)$ on the same axes.