Name:

Math 1830- Final Exam - December 14, 2006

Instructions: The exam is worth 150 points. Calculators are not permitted.

1. **(20 points)** Evaluate the following indefinite integrals:

   a. $\int t^3 + t - 5\, dt$

   b. $\int x\sqrt{1+2x^2}\, dx$

   c. $\int \sec x \tan x\, dx$

   d. $\int 3 + 2x(x^2-2)^5\, dx$

   e. $\int 6\, dx$.

2. **(15 points)** Evaluate the following definite integrals by any means you wish:

a. $\int_{-3}^{3} \sqrt{9 - v^2}\, dv$

b. $\int_{2}^{3} t(3 - t)^{1/3}\, dt$

c. $\int_{0}^{3} \mid x - 1 \mid\, dx$

3. **(10 points)** If oil leaks from a tank at a rate of $r(t)$ gallons per minute at time t, what does $\int_{0}^{120} r(t)\, dt$ represent?

4. (**15 points**) Evaluate the Riemann sum for

$$f(x) = x^2 - x \quad 0 \leq x \leq 3$$

with six equal intervals and taking your sample points to be the left endpoint of each interval. Explain, with the aid of a diagram, what the Riemann sum represents.

5. **(15 points)**

a. Find the area under the graph of $y = x^2 + 2$ and above the interval $[1, 2]$ on the x axis.

b. Let $f(x) = x^2$. Find the average value of f on the interval $[2, 5]$. Then find a value $c \in [2, 5]$ such that $f_{ave} = f(c)$.

6. **(20 points)** You are given $g(x)$. Find the derivative $g'(x)$:

a. $g(x) = x\sin(x)$

b. $g(x) = \frac{\sin x}{x^2+1}$

c. $g(x) = \int_1^x \sqrt{t^2 + \cos t}\, dt$

d. $g(x) = \int_1^{1/x} t^2 + t^3\, dt$

e. $g(x) = |x|$

7. **(10 points)** Use implicit differentiation to find the equation of the tangent line to the ellipse $x^2 + xy + y^2 = 3$ at the point $(1, 1)$.

8. **(10 points)** Evaluate:

a. $\lim_{t\to 0} \frac{\sin(4t)}{t}$

b. $2 - \frac{2}{9} + \frac{2}{27} - \frac{2}{81} + \frac{2}{243} \cdots$

c. $\lim_{n\to\infty} \sum_{i=1}^{n} \sin(\frac{\pi i}{2n}) \frac{\pi}{2n}$

d. $\lim_{x\to\infty} \frac{x^3+2x-1}{3x^4+6x^2-5x+12}$

9. **(15 points)** Below is sketched the graph of $y = f(x)$. Answer the following questions.

a. Find $\lim_{x \to 3^+} f(x)$.

b. Estimate $f'(5)$.

c. Estimate $f''(5)$.

d. Estimate the location of any inflection points.

e. At what x values does $f(x)$ fail to be differentiable?

f. Estimate $\int_0^3 f(x)\,dx$.

g. Find $\lim_{x \to 7} f(x)$

10. **(10 points)** *Using the definition* , show that the derivative of $f(x) = x^2$ is $f'(x) = 2x$.

11. **(10 points)** Prove that the equation $3 + x + 6x^3 + x^7 = 0$ has exactly one real root. Be clear about which theorems you cite in your proof, and why they apply.