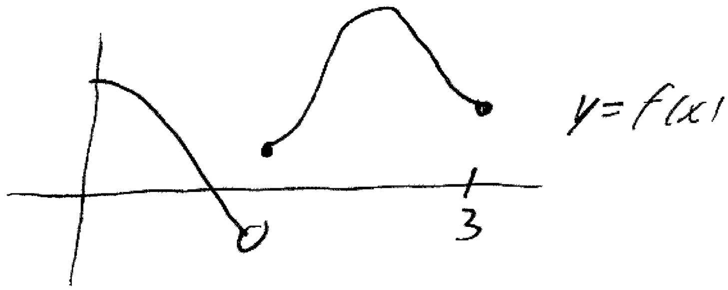
Name: SOLUTIONS

Math 1830- Midterm Exam #3 - November 20, 2006

1. **(10 points)** Sketch the graph of a function which has domain $[0, 3]$ and such that the function has a global maximum but no global minimum.



2. **(15 points)**

a. State the mean value theorem. Be sure to include all necessary hypotheses!

Suppose f(x) is continuous on [a,b] and differentiable on (a,b). Then $\exists$ $c \in (a,b)$ so

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

b. Let $f(x) = x^3 + x - 1$ on $[0, 2]$. Verify that $f(x)$ satisfies the hypotheses of the mean value theorem. Then find all numbers c that satisfy the conclusion of the mean value theorem.

f is a polynomial so it is cont & diff'ble on [0,2].

$$\frac{f(2)-f(0)}{2-0} = \frac{9 - -1}{2} = 5$$

$$f'(x) = 3x^2+1$$

$$3x^2+1 = 5$$

$$x = \pm 2/\sqrt{3} \qquad -2/\sqrt{3} \text{ not in } [0,2]$$

$$c = 2/\sqrt{3}$$

3. **(20 points)** Let $f(x) = 2x^{5/3} - 5x^{4/3}$. Then:

$$f'(x) = \frac{10}{3}x^{1/3}(x^{1/3} - 2), \quad f''(x) = \frac{20(x^{1/3} - 1)}{9x^{2/3}}.$$

Find all x and y intercepts, horizontal and/or vertical asymptotes, intervals where $f(x)$ is increasing or decreasing, concave up or concave down, and all local extrema and inflection points. Neatly sketch the graph of $y = f(x)$, labelling the x and y coordinates of the max/mins/inflection points/intercepts.

$2x^{5/3} - 5x^{4/3} = 0 \quad x^{4/3}(2x^{1/3} - 5) = 0 \quad x = 0$

$x^{1/3} = 5/2 \quad x = 125/8$

Domain $(-\infty, \infty)$

intercepts $(0,0)$, $(125/8, 0)$

No asymptotes

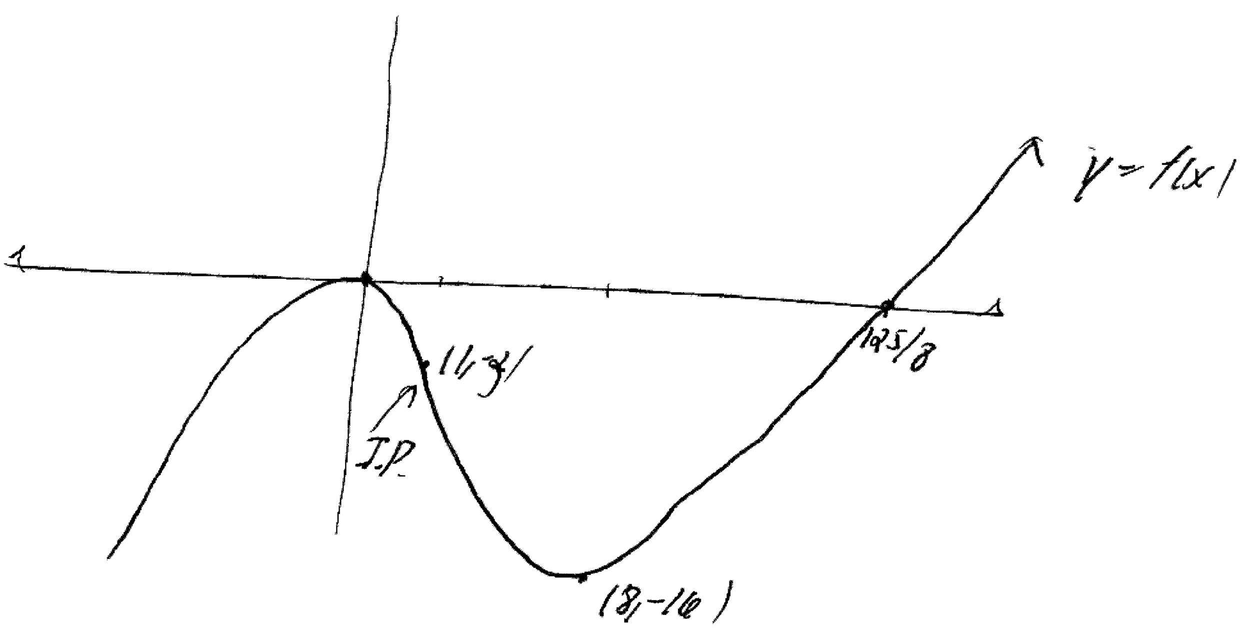
$f'(x)$: $+$ | 0 | $-$ | 8 | $+$

increasing $(-\infty, 0) \cup (8, \infty)$
decreasing $(0, 8)$

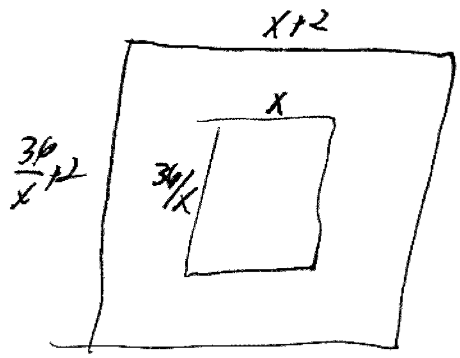
local max $(0,0)$
local min $(8, 2\cdot 32 - 5\cdot 16) = (8, -16)$

$f''(x)$: $-$ | 0 | $-$ | 1 | $+$

concave down $(-\infty, 0) \cup (0, 1)$
concave up $(1, \infty)$
I.P. $(1, -3)$



4. **(15 points)** A page will contain 36 square inches of print. It will have one inch margins on all four sides. Find the dimensions of the page so that the least amount of paper is used.



Minimize $(x+2)(\frac{36}{x}+2) = A(x)$

$A'(x) = (\frac{36}{x}+2) + (x+2)(-\frac{36}{x^2})$

$0 = \frac{36}{x}+2 - \frac{36}{x} - \frac{72}{x^2}$

$\frac{72}{x^2} = 2$

$x^2 = 36$

$x = 6$

$\boxed{8 \times 8 \text{ in}}$

5. **(10 points)** Find the most general antiderivative of the function $f(x)$:

a. $f(x) = 3x^2 + \sqrt{x}$

$x^3 + \frac{2}{3}x^{3/2} + C$

b. $f(x) = \cos x - 3\sin x$

$\sin x + 3\cos x + C$

c. $f(x) = 7$ $\quad 7x + C$

d. $f(x) = (x^2 + 2x + 5)^{20}(2x + 2)$

$\frac{1}{21}(x^2+2x+5)^{21} + C$

6. **(10 points)** Find the absolute maximum and absolute minimum values of $f(x) = 3x^2 - 12x + 5$ on the interval $[0, 3]$.

$f'(x) = 6x - 12$

$0 = 6x - 12 \qquad x = 2$

x	$f(x)$
0	5 ← global max value = 5
2	−7 ← global min value = −7
3	−4

7. **(10 points)** Let $f(x) = -3x^5 + 5x^3$.

a. Find all the critical numbers for $f(x)$.

$f'(x) = -15x^4 + 15x^2 = -15x^2(x^2 - 1) = -15x^4 + 15x^2$

$x = 0, 1, -1$

b. Use the *second derivative test* to classify, if possible, each critical number as a local maximum or local minimum.

$f''(x) = -60x^3 + 30x$

$f''(0) = 0$ no info

$f''(1) = -30$ local max

$f''(-1) = 30$ local min

8. **(10 points)** Find $f(x)$ given that $f''(x) = -x + 2$, $f'(1) = 1/2$, $f(0) = 3$.

$f'(x) = -\frac{1}{2}x^2 + 2x + C$

$\frac{1}{2} = -\frac{1}{2} + 2 + C \qquad C = -1$

$f'(x) = -\frac{1}{2}x^2 + 2x - 1$

$f(x) = -\frac{1}{6}x^3 + x^2 - x + D$

$3 = 0 + 0 + 0 + D$

$\boxed{f(x) = -\frac{1}{6}x^3 + x^2 - x + 3}$