

Name:

Math 1830- Midterm Exam #3 - November 20, 2006

1. (10 points) Sketch the graph of a function which has domain $[0, 3]$ and such that the function has a global maximum but no global minimum.

2. (15 points)

a. State the mean value theorem. Be sure to include all necessary hypotheses!

b. Let $f(x) = x^3 + x - 1$ on $[0, 2]$. Verify that $f(x)$ satisfies the hypotheses of the mean value theorem. Then find all numbers c that satisfy the conclusion of the mean value theorem.

3. (20 points) Let $f(x) = 2x^{5/3} - 5x^{4/3}$. Then:

$$f'(x) = \frac{10}{3}x^{1/3}(x^{1/3} - 2), \quad f''(x) = \frac{20(x^{1/3} - 1)}{9x^{2/3}}.$$

Find all x and y intercepts, horizontal and/or vertical asymptotes, intervals where $f(x)$ is increasing or decreasing, concave up or concave down, and all local extrema and inflection points. Neatly sketch the graph of $y = f(x)$, labelling the x and y coordinates of the max/mins/inflection points/intercepts.

4. **(15 points)** A page will contain 36 square inches of print. It will have one inch margins on all four sides. Find the dimensions of the page so that the least amount of paper is used.

5. **(10 points)** Find the most general antiderivative of the function $f(x)$:

a. $f(x) = 3x^2 + \sqrt{x}$

b. $f(x) = \cos x - 3 \sin x$

c. $f(x) = 7$

d. $f(x) = (x^2 + 2x + 5)^{20}(2x + 2)$

6. **(10 points)** Find the absolute maximum and absolute minimum values of $f(x) = 3x^2 - 12x + 5$ on the interval $[0, 3]$.

7. **(10 points)** Let $f(x) = -3x^5 + 5x^3$.

a. Find all the critical numbers for $f(x)$.

b. Use the *second derivative test* to classify, if possible, each critical number as a local maximum or local minimum.

8. **(10 points)** Find $f(x)$ given that $f''(x) = -x + 2$, $f'(1) = 1/2$, $f(0) = 3$.