Name:

Math 141- Final Exam - December 14, 2007

Instructions: The exam is worth 150 points. Calculators are not permitted.

1. **(15 points)** Evaluate the following indefinite integrals:

    a. $\int e^x \sin(e^x)\ dx$

    b. $\int \sqrt{x} + \frac{1}{x}\ dx$

    c. $\int \frac{1}{1+x^2}\ dx$

    d. $\int \frac{x}{1+x^2}\ dx$

2. **(10 points)** Consider the definite integral $\int_1^3 2x+1\ dx$.

a. Estimate it with a Riemann sum with 6 equal intervals and the right hand endpoints.
b. Write the Riemann sum corresponding to n equal intervals, again using right endpoints.
c. Let $n \to \infty$ to get the actual value of the integral.

You may use the formula

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

d. Check your answer by evaluating the integral with the fundamental theorem of calculus.

3. **(10 points)** Evaluate the following definite integrals by any means you wish (i.e. using FTOC or areas etc...):

a. $\int_{-2}^{2} \sqrt{4 - x^2}\, dx$

b. $\int_{e}^{e^4} \frac{dx}{x\sqrt{\ln x}}$

c. $\int_{0}^{5} x(x^2 + 1)^{15} dx$

4. **(10 points)** Let

$$f(x) = \int_1^x \frac{t}{1+t+t^2}\, dt.$$

a. What is $f'(x)$.

b. On which intervals is $f(x)$ increasing/ decreasing?

5. **(5 points)** The velocity function of a particle moving along a line is given in meters per second by $v(t) = 3t - 5$ for $0 \leq t \leq 3$. Find the total distance the particle traveled during the time interval.

6. **(5 points)** If oil leaks from a tank at a rate of $r(t)$ gallons per minute at time t minutes, what does $\int_0^{60} r(t)dt$ represent?

7. **(10 points)** Find the area under the graph of $y = \sin(2x)$ and above the interval $[0, \pi/2]$ on the x axis.

8. **(5 points)** State precisely the intermediate value theorem, including any necessary hypotheses.

9. **(10 points)** Find the equation of the tangent line to the graph of $y = x^2 + 2x + 1$ at the point where $x = 2$.

10. **(5 points)** Sketch the graph of a function which is continuous but not differentiable at $x = 2$.

11. **(20 points)** Find $\frac{dy}{dx}$

a. $y = x\cos(x)$

b. $y = \frac{x}{x^2+1}$

c. $y = \int_1^x \sqrt{t^2+\cos t}\,dt$

d. $y = \tan(e^{2x})$

e. $\ln(y) + xy = 3$

12. **(15 points)** Below is sketched the graph of $y = f(x)$. Answer the following questions.

a. Find $\lim_{x\to 3^+} f(x)$.

b. Estimate $f'(5)$.

d. Estimate the location of any inflection points.

e. At what x values does $f(x)$ fail to be differentiable?

f. Estimate $\int_0^3 f(x)\,dx$.

g. Find $\lim_{x\to 7} f(x)$

13. **(10 points)** Let $f(x) = x^2 - 2x + 3$. Find the global maximum and minimum values of $f(x)$ on the interval $[-1, 3]$.

14. **(10 points)** Evaluate the following limits, if they exist:

$$a.\ \lim_{x\to\infty} \frac{x^3 + x + 1}{x^4 + 2x + 3} \qquad b.\ \lim_{x\to 3^+} \frac{x^2 - 9}{x^2 - 2x - 3} \qquad c.\ \lim_{h\to 0} \frac{\sqrt[4]{16 + h} - 2}{h}.$$

15. **(10 points)** Let $f(x) = 4x^3 + 3x^2 - 6x + 1$. Find the intervals on which f is increasing or decreasing. Find the local maximum and minimum values of f. Find the intervals of concavity and inflection points. Then neatly sketch the graph $y = f(x)$.