

Name:

SOLUTIONS

Math 141- Midterm Exam #3 - November 12, 2007

1. (20 points) Let $f(x) = \frac{x}{x^2+1}$. Find the global maximum and global minimum values of $f(x)$ on the interval $[0, 2]$.

$$f' = \frac{x^2+1 - x \cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} \quad \text{C.P. } x = \pm 1$$

x	$f(x)$
0	0 \leftarrow Global min value = 0
1	$1/2$ \leftarrow Global max value = $1/2$
2	$2/5$

2. (5 points) Complete the following definition. A function $f(x)$ has a *local minimum* at $x = c$ if \dots .

there is $\epsilon > 0$ so

$$f(x) \geq f(c) \text{ for all } x \text{ in } (c-\epsilon, c+\epsilon)$$

3. (15 points)

Let $f(x) = \frac{x}{x+2}$. Verify that $f(x)$ satisfies the hypotheses of the Mean Value Theorem on the interval $[1, 4]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$f(x)$ is continuous on $[1, 4]$ and differentiable on $(1, 4)$
so MVT applies

$$\frac{f(4) - f(1)}{4 - 1} = \frac{\frac{2}{3} - \frac{1}{3}}{3} = \frac{1}{9}$$

$$f'(x) = \frac{x+2 - x}{(x+2)^2} = \frac{2}{(x+2)^2}$$

Set $\frac{2}{(x+2)^2} = \frac{1}{9}$

$$18 = (x+2)^2$$

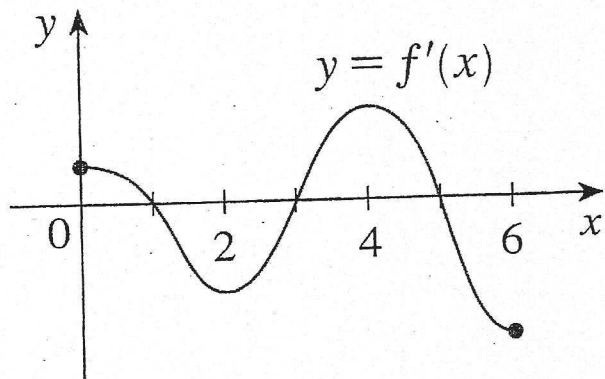
$$x+2 = \pm 3\sqrt{3}$$

$$x = -2 \pm 3\sqrt{3}$$

$$\boxed{c = -2 + 3\sqrt{3}}$$

$-2 - 3\sqrt{3}$ not in $(1, 4)$

4. (15 points) The graph of the derivative f' of a function f is shown.
- On what intervals is f increasing or decreasing?
 - At what values of x does f have a local maximum or minimum?
 - At what values of x does the graph of $f(x)$ have inflection points?



- a. Increasing $(0, 1) \cup (3, 5)$
Decreasing $(1, 3) \cup (5, 6)$
- b. Local max at $x=1, 5$
Local min at $x=3$
- c. Inflection points at $x=2, 4$

5. (20 points) Let

$$f(x) = (x^2 - 1)^{2/3}.$$

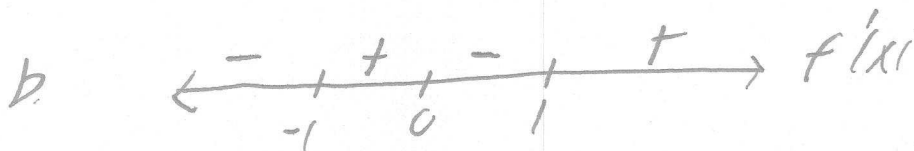
Then:

$$f'(x) = \frac{4}{3} \frac{x}{(x^2 - 1)^{1/3}}, \quad f''(x) = \frac{4}{9} \frac{(x^2 - 3)}{(x^2 - 1)^{4/3}}.$$

- Find all x and y intercepts and any asymptotes.
- Find the intervals where $f(x)$ is increasing or decreasing and any local maximums or local minimums.
- Find the intervals where $f(x)$ is concave up or concave down, and determine any inflection points.
- Neatly sketch the graph of $y = f(x)$, Label the x and y coordinates of any intercepts, local extrema and inflection points.

a. $(0, 1)$ $(1, 0)$ $(-1, 0)$

~~V.A. $x=1$, $x=-1$~~ No H.A., No V.A.



Decreasing $(-\infty, -1)$ $(0, 1)$

Increasing $(-1, 0)$ $(1, \infty)$

Local min $(-1, 0)$, $(1, 0)$

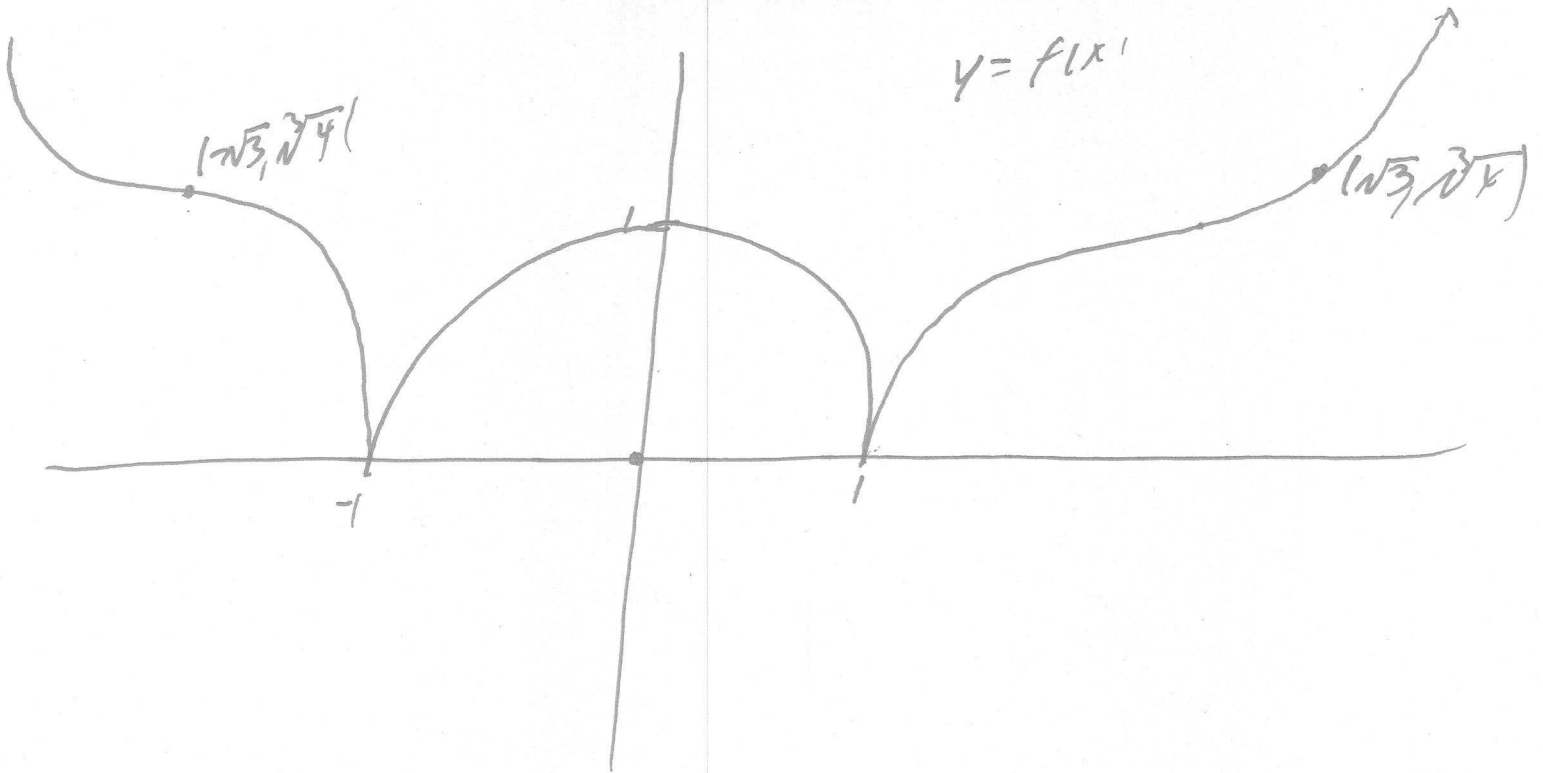
Local max $(0, 1)$



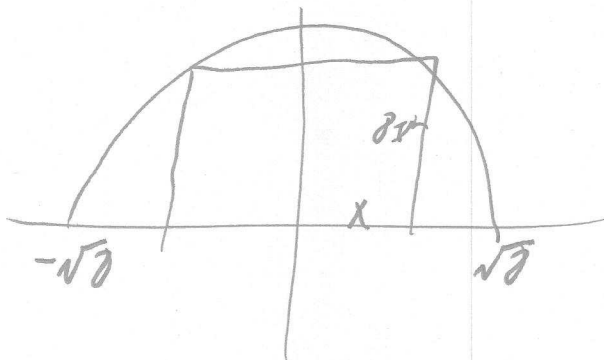
Concave up $(-\infty, -\sqrt{3})$ $(\sqrt{3}, \infty)$

Concave down $(-\sqrt{3}, -1)$ $(-1, 1)$ $(1, \sqrt{3})$

I.P. $(-\sqrt{3}, \sqrt[3]{4})$, $(\sqrt{3}, \sqrt[3]{4})$



6. (15 points) Find the dimensions of the rectangle of largest area that has its base on the x -axis and its other two vertices above the x -axis and lying on the parabola $y = 8 - x^2$.



$$\text{Maximize } A(x) = 2x(8 - x^2) \quad 0 \leq x \leq \sqrt{8}$$

$$= 16x - 2x^3$$

$$A'(x) = 16 - 6x^2$$

$$x = \pm \sqrt{\frac{16}{6}} = \sqrt{\frac{8}{3}}$$

$$x = \sqrt{\frac{8}{3}} \quad y = 8 - \frac{8}{3} = \frac{16}{3}$$

Dimensions $\boxed{2\sqrt{\frac{8}{3}} \times \frac{16}{3}}$

7. (10 points) Evaluate the following limits:

a. $\lim_{x \rightarrow 0^+} \frac{\cos x}{x}$,

b. $\lim_{x \rightarrow \infty} (e^x + x)^{1/x}$

a. (∞) (L'Hôpital does not apply)

b. $y = (e^x + x)^{1/x}$

$$\ln y = \frac{1}{x} \ln |e^x + x|$$

$$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \frac{\ln |e^x + x|}{x} \stackrel{\text{L'Hôpital}}{=} \lim_{x \rightarrow \infty} \frac{e^x + 1}{e^x + x}$$

$$\stackrel{\text{L'Hôpital}}{=} \lim_{x \rightarrow \infty} \frac{e^x}{e^x + 1}$$

$$\stackrel{\text{L'Hôpital}}{=} \lim_{x \rightarrow \infty} \frac{e^x}{e^x} = 1$$

Thus $\lim_{x \rightarrow \infty} y = e^1 = \textcircled{e}$