Name:

Math 141- Midterm Exam #3 - November 12, 2007

1. **(20 points)** Let $f(x) = \frac{x}{x^2+1}$. Find the global maximum and global minimum values of $f(x)$ on the interval $[0, 2]$.

2. **(5 points)** Complete the following definition. A function $f(x)$ has a *local minimum* at $x = c$ if $\cdots$.

3. **(15 points)**
Let $f(x) = \frac{x}{x+2}$. Verify that $f(x)$ satisfies the hypotheses of the Mean Value Theorem on the interval $[1, 4]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

4. **(15 points)** The graph of the *derivative* f' of a function f is shown.
   a. On what intervals is f increasing or decreasing?
   b. At what values of x does f have a local maximum or minimum?
   c. At what values of x does the graph of $f(x)$ have inflection points?

5. (**20 points**) Let

$$f(x) = (x^2 - 1)^{2/3}.$$

Then:

$$f'(x) = \frac{4}{3}\frac{x}{(x^2-1)^{1/3}}, \qquad f''(x) = \frac{4}{9}\frac{(x^2-3)}{(x^2-1)^{4/3}}.$$

a. Find all x and y intercepts and any asymptotes.

b. Find the intervals where $f(x)$ is increasing or decreasing and any local maximums or local minimums.

c. Find the intervals where $f(x)$ is concave up or concave down, and determine any inflection points.

d. Neatly sketch the graph of $y = f(x)$, Label the x and y coordinates of any intercepts, local extrema and inflection points.

6. **(15 points)** Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 8 - x^2$.

7. **(10 points)** Evaluate the following limits:

$$a.\ \lim_{x \to 0^+} \frac{\cos x}{x}, \qquad\qquad b.\ \lim_{x \to \infty} (e^x + x)^{1/x}$$