

SOLUTIONS

Math 141A- Midterm Exam #1 - September 25, 2014

1. (15 points) True or false:

- T a. A function which is differentiable at $x = a$ must also be continuous at $x = a$.
- T b. The composition of two continuous functions is also continuous.
- F c. Suppose $f(x)$ is continuous on $[0, 2]$ and $f(0) = 2, f(2) = 5$. Then the intermediate value theorem implies $f(x)$ does not have a root in $(0, 2)$.
- F d. Suppose $y = a$ is a horizontal asymptote for $f(x)$. Then the graph of $y = f(x)$ does not cross the line $y = a$.
- T e. $f(x) = \frac{\sin x}{x}$ has a removable discontinuity at $x = 0$.

2. (20 points)

a. Give the $\epsilon - \delta$ definition for $\lim_{x \rightarrow a} f(x) = L$.

For any $\epsilon > 0$ there is a δ such that
if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

b. Use the definition to prove that

$$\lim_{x \rightarrow 3} (2x + 8) = 14.$$

Let $\epsilon > 0$ be given. Choose $\delta = \epsilon/2$. Suppose
 $0 < |x - 3| < \delta$. We must prove $|f(x) - L| < \epsilon$ where
 $f(x) = 2x + 8$ and $L = 14$. But

$$|f(x) - L| = |2x + 8 - 14| = |2x - 6| = 2|x - 3| < 2\delta = \epsilon.$$

So $|f(x) - L| < \epsilon$ as desired. //

3. (20 points) Evaluate the following limits. If the limit does not exist then write DNE.

a. $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+5}}{2x-3}$ Since $x < 0$ $x = -\sqrt{x^2}$ Divide top & bottom by x

$\lim_{x \rightarrow -\infty} \frac{-\sqrt{1 + \frac{5}{x^2}}}{2 - \frac{3}{x}} = \left(-\frac{1}{2}\right)$

b. $\lim_{x \rightarrow 3^+} \frac{5-3x}{(x-3)(x-5)}$

Try $x = 3.001$

$\approx \frac{-4}{(001)(-2)}$

∞

c. $\lim_{x \rightarrow 6} \sin x$.

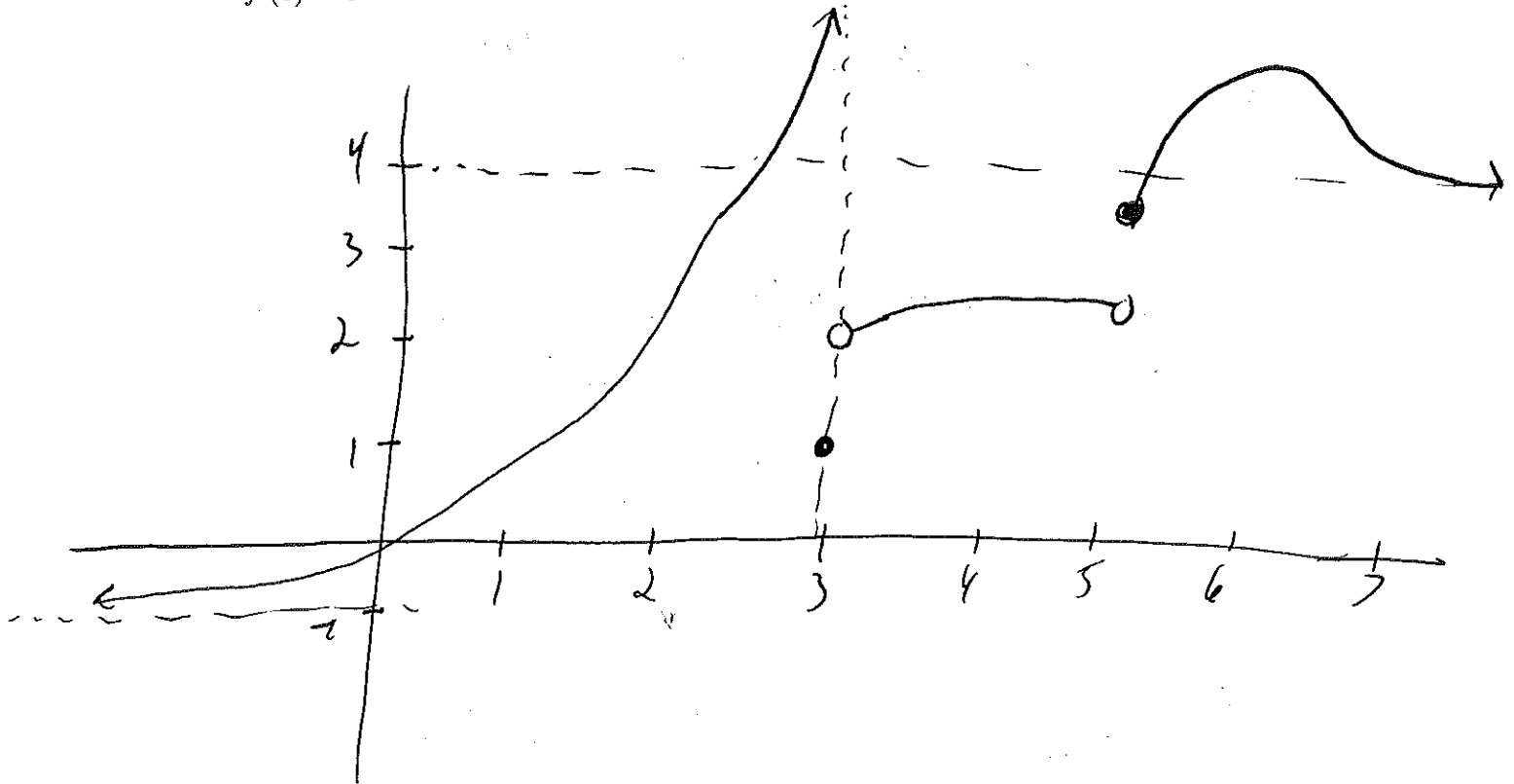
$\sin 6$

d. Suppose $\lim_{x \rightarrow 2} f(x) = 3$ and $\lim_{x \rightarrow 2} g(x) = -1$. Evaluate $\lim_{x \rightarrow 2} \frac{f(x) + 3(g(x)^2)}{\sqrt{f(x)}}$.

$\frac{6}{\sqrt{3}}$

4. (10 points) Neatly sketch the graph of a single function $f(x)$ which has the following properties:

- $\lim_{x \rightarrow 3^+} f(x) = 2$, $\lim_{x \rightarrow 3^-} f(x) = \infty$, $f(3) = 1$.
- $f(x)$ is continuous from the right at $x = 5$ but not continuous from the left at $x = 5$.
- $\lim_{x \rightarrow \infty} f(x) = 4$, $\lim_{x \rightarrow -\infty} f(x) = -1$.
- $f'(6) = 0$.



5. (20 points) A ball is tossed and has height in feet given at time t seconds by $y(t) = -t^2 + 6t$.

a. Use the definition of the derivative to prove that $y'(t) = -2t + 6$.

$$\begin{aligned}
 y'(t) &= \lim_{h \rightarrow 0} \frac{y(t+h) - y(t)}{h} = \lim_{h \rightarrow 0} \frac{-(t+h)^2 + 6(t+h) - (-t^2 + 6t)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-t^2 - 2th - h^2 + 6t + 6h + t^2 - 6t}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-2th + 6h - h^2}{h} = \lim_{h \rightarrow 0} (-2t + 6 - h) \\
 &= \boxed{-2t + 6}
 \end{aligned}$$

b. Find the equation of the tangent line to $y = y(t)$ at the point where $t = 2$.

point $(2, 8)$

slope $= y'(2) = 2$

$$\boxed{y - 8 = 2(t - 2)}$$

c. What is the ball's average velocity from time $t = 1$ to time $t = 3$?

$$\frac{y(3) - y(1)}{3 - 1} = \frac{9 - 5}{3 - 1} = \boxed{2 \text{ ft/sec}}$$

d. How fast is the ball moving when it hits the ground?

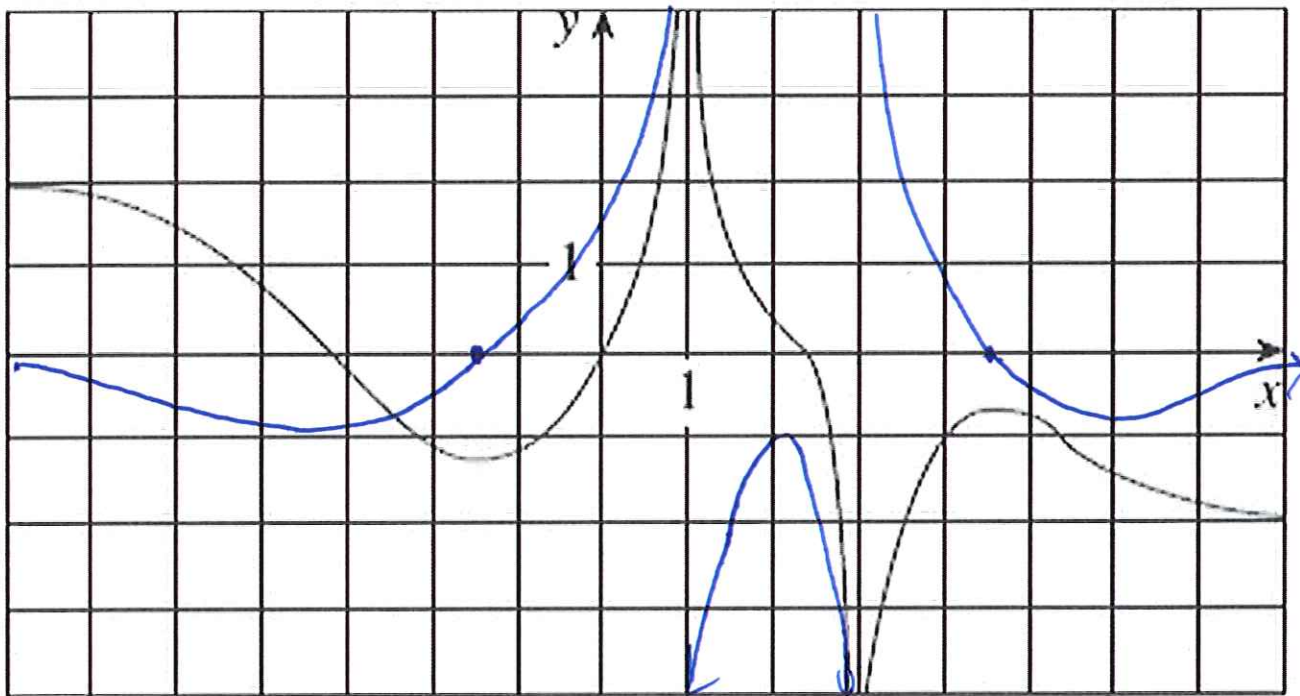
$$-t^2 + 6t = 0 \quad t = 0, 6$$

$$y'(6) = -12 + 6 = -6$$

going 6 ft/sec
down

e. What is the ball's acceleration?

$$\text{accel} = y'' = \boxed{-2 \text{ ft/sec}^2}$$



6. (15 points) Above is the graph of a function $y = f(x)$.

a. Find the vertical and horizontal asymptotes.

$$\begin{array}{l} x=1 \quad x=3 \quad \text{V.A.} \\ y=2 \quad y=-2 \quad \text{H.A.} \end{array}$$

b. Evaluate $\lim_{x \rightarrow \infty} f(x)$.

$$-2$$

c. Estimate $f'(0)$.

$$2$$

d. Estimate $\lim_{x \rightarrow \infty} f'(x)$.

$$0$$

e. On the same axes above carefully sketch a graph of $y = f'(x)$

Name: