Math 141A Fall 2014-Review Sheet for 3rd Midterm

The third midterm is Monday 11/17/2014 and will cover Chapter 4, excluding 4.6 This sheet is designed to help you organize your studying and is not exhaustive! Remember no calculators or notes are allowed.

Definitions to know: Make sure you learn the precise definitions not just vague statements like "critical numbers are where max and mins occur." All these definitions are in boldface at various points in the chapter.

Absolute/global maximum/minimum
extreme value
local maximum/minimum
critical number
increasing/decreasing
inflection point
marginal cost,
even/odd function
antiderivative
position/velocity/acceleration

Theory to know: Below are the theorems you should know with brief summaries of them, be sure to learn the actual statement from the book. Also try to know some examples of how the theorems can fail when the hypotheses do not hold, for example why does the MVT require continuous function on a closed interval.

- **Extreme value theorem:** Says that continuous functions on closed intervals attain an absolute max and min value inside the interval.
- **Fermat's Theorem**: local extrema occur at critical numbers.
- **Mean value theorem**: For a continuous function on a closed interval which is differentiable there is some point c in (a,b) with f'(c) equal to the average rate of change of f over the interval. Be sure to understand this theorem graphically as well (e.g. p.285 figure 3,4)
- **Rolle's theorem**: special case of MVT when f(a)=f(b)=0.
- **Corollaries to MVT** on p. 284: If f'(x)=0 on an interval I then the function is constant on I. Thus two functions with the same derivative on I differ only be a constant on I.
- **L'Hospital's rule**: Be able to do all the various indeterminate forms.
- **Newton's Method**

Curve sketching

- Figure out "non-calculus" information about a curve, namely x and y intercepts, horizontal and vertical asymptotes, and whether the function is even/odd/neither or periodic.

- Use the first derivative to determine behavior of the function. Specifically:
    1. Where is the function increasing/decreasing.
    2. First derivative test to classify critical numbers as local max, local min or neither.
- Use the second derivative to determine the behavior of the function. Specifically:
    1. Where is the function concave up or concave down.
    2. What are the inflection points if any.
    3. Second derivative test.

- Put all this information together to make a sketch of the graph of y=f(x).
- Given information about a curve, sketch it's graph. (e.g. 4.1 #7-10, etc..)
- Given graphs of f(x) sketch f'(x) and f''(x) and vice versa, i.e. given f'(x) sketch a possible graph of f(x) and f''(x) etc..

Types of problems: The homework assignments and to a lesser extent the quizzes are the best place to look for problem types. The list below is not complete. In particular each part of the curve sketching above can be a single problem, i.e. "Use the second derivative test to classify the local max/mins of the function below" or "Determine on which intervals f(x) is concave up or concave down."

- Illustrate or use theorems, such as "Find a point c in (a,b) which satifies the conclusions of the MVT." Or "Use the MVT to prove that y=f(x) has exactly one real root.
- Illustrate why hypotheses are necessary for the theorems, e.g. "Give an example of a function which is not continuous on a closed interval and such that the extreme value theorem does not hold." Or "Show that no c works and explain why this does not contradict the MVT."
- All the information in the curve sketching can also be tested graphically, i.e. you are given a graph of the derivative f'(x) and asked to find out all the information about f(x) and sketch.
- Find global extrema of a continuous function on a closed interval.
- Optimization problems: These are each a little different so be sure to practice and learn the general procedure outlined on p. 325-6. Understand how the 1st derivative test can sometimes guarantee a *global* extreme instead of just local (in box on p.328).
- Use L'Hospital's rule to evaluate limits, including indeterminate forms that need some manipulation before L'Hospital's rule applies, for example taking natural log first to evaluate indeterminate powers.
- Use Newton's method to approximate roots of a polynomial
- Calculate antiderivatives of elementary functions. Use to solve problems involving rectilinear motion.