Name: SOLUTIONS

Math 1830- Midterm Exam #2 - October 20, 2006

1. **(30 points)** Find $\frac{dy}{dx}$:

a. $y = \csc(x)$

$$y' = -\csc x \cot x$$

b. $y = \frac{x\sin(x)}{2x^2-3}$

$$y' = \frac{(2x^2-3)(x\cos x + \sin x) - (x\sin x)(4x)}{(2x^2-3)^2}$$

c. $y = \sqrt{\tan(x) + x^3}$

$$y' = \frac{1}{2\sqrt{\tan x + x^3}} \cdot (\sec^2 x + 3x^2)$$

d. $\sin(xy) = 2x^3 - y^2$

$$\cos(xy)(y + xy') = 6x^2 - 2yy'$$

$$y'(x\cos(xy) + 2y) = 6x^2 - y\cos(xy)$$

$$\boxed{y' = \frac{6x^2 - y\cos(xy)}{x\cos(xy) + 2y}}$$

e. $y = \sin(\sin(x^2))$

$$y' = \cos(\sin(x^2)) \cdot \cos(x^2) \cdot 2x$$

f. $y = (x^2 + 2x + 3)^{10}(6x^3 - \frac{1}{x})^5$

$$y' = 10(x^2+2x+3)^9(2x+2)(6x^3 - \tfrac{1}{x})^5$$

$$+$$

$$(x^2+2x+3)^{10} \cdot 5(6x^3 - \tfrac{1}{x})^4(18x^2 + \tfrac{1}{x^2})$$

2. **(10 points)** Find y'' *by implicit differentiation.* Your final answer should be in terms of x and y. and y'.

$$xy + \frac{1}{y} = 1$$

$$y + xy' - \frac{1}{y^2} \cdot y' = 0$$

$$\frac{1}{y^2}y' = y + xy'$$

$$y' = \frac{y}{\frac{1}{y^2} - x} = \frac{y^3}{1 - xy^2}$$

$$y'' = \frac{(1-xy^2)(3y^2y') - y^3(-y^2 - 2xyy')}{(1-xy^2)^2}$$

3. **(12 points)** A table of values for f, g, f', and g' is given:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

a. If $h(x) = f(g(x))$, find $h'(1)$.

$f'(g(1)) \cdot g'(1) = f'(2) \cdot 6 = 30$

b. If $r(x) = g(f(x))$, find $r'(1)$.

$g'(f(1)) \cdot f'(1) = g'(3) \cdot 4 = 36$

c. If $v(x) = f(x)g(x)$ find $v'(2)$

$f(2)g'(2) + f'(2)g(2) = 1 \cdot 7 + 5 \cdot 8 = 47$

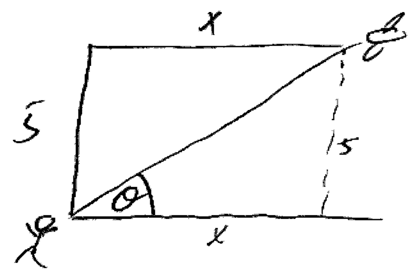
d. If $u(x) = \frac{f(x)}{g(x)}$ find $u'(3)$.

$$\frac{g(3)f'(3) - f(3)g'(3)}{g(3)^2} = \frac{2 \cdot 7 - 7 \cdot 9}{4} = \frac{-49}{4}$$

4. **(8 points)** State three ways a function $y = f(x)$ can fail to be differentiable at $x = a$.

- vertical tangent line
- corner in graph
- f is discontinuous at $x = a$

5. **(15 points)** An airplane files at at altitude of 5 miles toward a point directly over an observer. The speed of the plane is 600 miles per hour. Find the rate at which the angle of elevation is changing when this angle is ~~30~~ 45 degrees.



Given $\frac{dx}{dt} = -600$ mph Find $\frac{d\theta}{dt}$ when $\theta = 45°$.

$$\tan\theta = \frac{5}{x}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{-5}{x^2}\frac{dx}{dt}$$

When $\theta = \pi/4$ Then $\cos\theta = \frac{\sqrt{2}}{2}$ so $\sec^2\theta = \left(\frac{2}{\sqrt{2}}\right)^2 = 2$

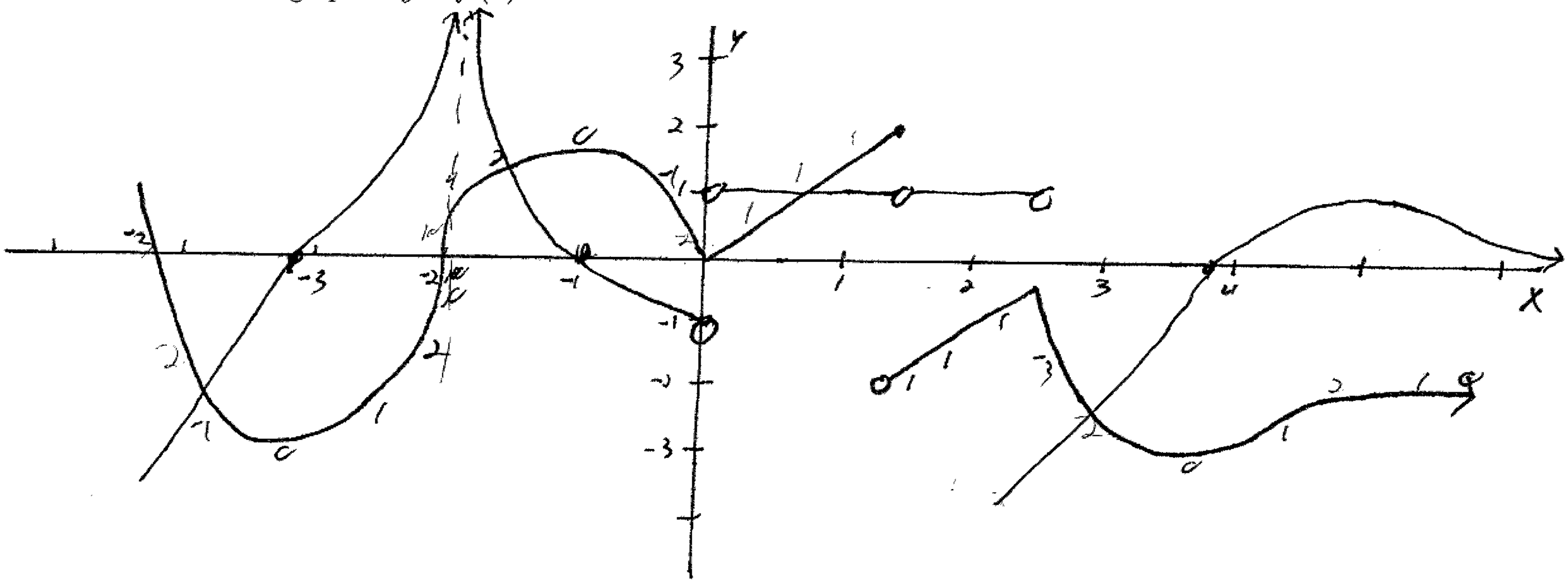
Also $x = 5$.

Thus

$$2 \cdot \frac{d\theta}{dt} = \frac{-5}{25} \cdot (-600)$$

$$\frac{d\theta}{dt} = \frac{-5}{50} \cdot -600 = 60 \text{ rad}/\cancel{\text{sec}}\ \text{hour}$$

6. (**15 points**) Below is sketched the graph of a function $y = f(x)$. On the same axes sketch the graph of $y = f'(x)$.



7. (**10 points**) Find the equation of the tangent line to the graph of the equation below at the point $(1, 2)$:

$$(x^2 + y^2)^4 + 2x = xy + 625$$

$$4(x^2+y^2)^3(2x+2yy') + 2 = xy' + y$$

$$4(5)^3(2+4y') + 2 = y' + 2$$

$$500(2+4y') + 2 = y' + 2$$

$$1000 + 2000y' + 2 = y' + 2$$

$$1999y' = -1000$$

$$y' = -1000/1999$$

$$y - 2 = -1000/1999\,(x - 1)$$