SOLUTIONS

Math 141- Midterm Exam #2 - October 22, 2014

1. **(50 points)** Find $\frac{dy}{dx}$. You do not need to simplify your answers.

a. $y = xe^{\cos x}$

$$y' = e^{\cos x} - x\sin x\, e^{\cos x}$$

b. $y = \log_3(x)$

$$y' = \frac{1}{x\ln 3}$$

c. $y = \ln(x^2 + 2x + 1)$

$$y' = \frac{2x+2}{x^2+2x+1}$$

d. $y = \frac{\sin x}{e^x}$

$$y' = \frac{e^x\cos x - e^x\sin x}{e^{2x}}$$

$$= \frac{\cos x - \sin x}{e^x}$$

e. $y = x^{\sec x}$

$$\ln y = \ln(x^{\sec x}) = \sec x \ln x$$

$$\frac{1}{y} y' = \sec x \tan x \ln x + \frac{\sec x}{x}$$

$$y' = x^{\sec x}\left(\sec x \tan x \ln x + \frac{\sec x}{x}\right)$$

f. $y = x^2 \sin x \cos x$

$$y' = 2x \sin x \cos x + x^2 \cos^2 x - x^2 \sin^2 x$$

g. $y = \sqrt{2 + \tan(1 + x^3)}$

$$y' = \frac{1}{2}(2 + \tan(1+x^3))^{-1/2}(\sec^2(1+x^3))(3x^2)$$

h. $xy^2 + 5x^2 - 2y = 10$

$$y^2 + 2xyy' + 10x - 2y' = 0$$

$$y'(2xy - 2) = -10x - y^2$$

$$y' = \frac{y^2 + 10x}{2 - 2xy}$$

i. $y = \sqrt{\frac{(x^2+1)^5 e^x}{x^2+2}}$

$$\ln y = \frac{1}{2}\left(5\ln(x^2+1) + x - \ln(x^2+2)\right)$$

$$\frac{1}{y}y' = \frac{1}{2}\left(\frac{10x}{x^2+1} + 1 - \frac{2x}{x^2+2}\right)$$

$$y' = \left(\sqrt{\frac{(x^2+1)^5 e^x}{x^2+2}}\right) \cdot \frac{1}{2} \cdot \left(\frac{10x}{x^2+1} + 1 - \frac{2x}{x^2+2}\right)$$

j. $y = \sin^{-1}(4x)$.

$$y' = \frac{4}{\sqrt{1 - 16x^2}}$$

2. **(10 points)** Suppose I deposit \$1000 in a bank account with continuously compounding interest. After three years time I now have \$1300. What is the annual rate *in percent*? (it is ok to leave your answer in terms of ln.)

$$P(t) = P_0 e^{kt} = 1000e^{kt}$$

$$1300 = P(3) = 1000e^{3k}$$

$$1.3 = e^{3k}$$

$$\ln(1.3) = 3k$$

$$k = \frac{\ln(1.3)}{3}$$

in percent need $\cdot 100$

$$\frac{100\ln(1.3)}{3}\%$$

3. **(10 points)** Estimate ln(0.99) using a linear approximation to an appropriate function.

$f(x) = \ln x$ use lin approx at $a = 1$

$f'(x) = \frac{1}{x}$ $\quad f(a) = \ln 1 = 0$

$f'(a) = 1$

$$L(x) = f(a) + f'(a)(x-a)$$

$$= 0 + 1(x-1) = x-1$$

$$L(.99) = .99 - 1 = -.01$$

4. **(10 points)** Suppose $xy + e^y = e$.

a. Find the equation of the tangent line at the point on the curve where $x = 0$.

b. Find y'' at that same point.

$$y + xy' + e^y y' = 0$$

$$y' = \frac{-y}{x+e^y}$$

$$y'' = \frac{(x+e^y)(-y') + y(1+e^y y')}{(x+e^y)^2}$$

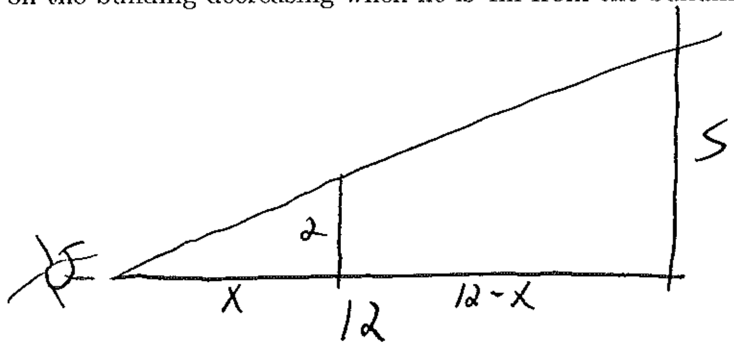
a. At $x=0$ $y=1$ $\quad y' = \frac{-1}{0+e} = -1/e$

$$\boxed{y-1 = -\frac{1}{e}(x-0)}$$

b. Plug in $x=0$ $y=1$ $y' = -1/e$ into y'':

$$y'' = \frac{e(1/e) + (1+e(-1/e))}{e^2} = \frac{1+1-1}{e^2} = 1/e^2$$

5. **(10 points)** A spotlight on the ground shines on a wall 12m away. If a man 2m tall walks from the spotlight toward the builing at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4m from the building? (Hint: use similar triangles)



Given $\frac{dx}{dt} = 1.6 \, m/s$ Find $\frac{ds}{dt}$ when $X = 8$

Similar triangles gives $\frac{S}{12} = \frac{2}{X}$

$SX = 24$

$S \cdot dx/dt + \frac{ds}{dt} \cdot X = 0$

When $X = 8 \Rightarrow \frac{S}{12} = \frac{2}{8} \Rightarrow S = 3$

$3 \cdot (1.6) + \frac{ds}{dt}(8) = 0$

$\frac{ds}{dt} = \frac{-4.8}{8} = -0.6 \, m/s$

Shadow is decreasing at $-0.6 \, m/s$

6. **(10 points)** The equation $x^2 - xy + y^2 = 3$ represents a "rotated ellipse", that is an ellipse whose axes are not parallel to the coordinate axes. Find the points where this ellipse intersects the x axis (you should get two points). Show that the tangent lines at these points are parallel.

intersects x axis means $y=0$ so $x^2=3$ $x=\pm\sqrt{3}$

$\boxed{\text{points } (\sqrt{3},0) \quad (-\sqrt{3},0)}$

Find y':

$$2x - xy' - y + 2yy' = 0$$

$$y' = \frac{y-2x}{2y-x}$$

at $(\sqrt{3},0)$ $\quad y' = \frac{0-2\sqrt{3}}{-\sqrt{3}} = 2$

at $(-\sqrt{3},0)$ $\quad y' = \frac{0+2\sqrt{3}}{0+\sqrt{3}} = 2$

tangent lines have same slope so parallel