MTH444/MTH544 **MIDTERM EXAM** **Name:**

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Read instructions carefully. For full credit, explain the main steps in your calculations.
MTH444 students: answer two of the three questions.
MTH544 students: answer all three questions.

1. Consider the continuity equation

$$\frac{\partial \rho}{\partial t} + C(\rho)\frac{\partial \rho}{\partial x} = 0$$

where $C(\rho) = \rho = J'(\rho)$ and $J(\rho) = \rho F(\rho)$. Assume that the initial condition is

$$\rho(x,0) = \begin{cases} 1 & x \le -1 \\ (1-x)/2 & -1 \le x \le 1 \\ 0 & x \ge 1 \end{cases}$$

(a) Find the characteristics and sketch them in the x-t plane.

(b) Show that a shock wave develops at time t_s and find t_s. In addition, find the location and velocity of the shock wave as functions of time.

(c) Find the solution to the PDE.

(d) Sketch the solution to the PDE as a function of x for t=1 and t=4. Explain your answer.

2. Consider a cylinder of constant cross section σ, whose axis lies on the x-axis. Assume that the cylinder's segment located at $A_L \le x \le A_R$ at t=0, occupies the interval $\alpha(t) \le x \le \beta(t)$ for $t>0$. In addition, assume that the total mass of this cylinder's segment is constant.

(a) Derive the continuity equation for the mass density ρ(x,t) in spatial coordinates. Explain all steps in your derivation.

(b) Use the result in part (a) to derive the continuity equation for the mass density R(A,t) in material coordinates. In addition, find the solution to this equation.

3. A linearly elastic bungee cord is hung vertically with the upper end fixed and the lower end free. The initial length is L_0 and the initial density is $R(A,0) = \alpha(1+A/L_0)$. Assume that the only external force acting on the cord is due to gravity.

After being hung, the cord stretches to length L and reaches a steady state.

(a) Find the stress T in steady state as a function of material coordinate A.

(b) Find the length of the cord L in steady state.