(a) $(1+t)\dfrac{dy}{dt} = 2(1+t)t^2y^2 + y$

$$\frac{dy}{dt} - \frac{y}{1+t} = 2t^2y^2$$

A Bernoulli equation with $n=2$, $p(t) = \dfrac{-1}{1+t}$, $q(t) = 2t^2$.

Use the substitution: $v = y^{1-n} \overset{\checkmark}{=} \dfrac{1}{y}$

ODE for v: $\dfrac{dv}{dt} + (-1)\cdot\left(\dfrac{-v}{1+t}\right) = (-1)\cdot 2t^2$

$$\Rightarrow \frac{dv}{dt} + \frac{v}{1+t} = -2t^2$$

A linear ODE with $g(t) = \dfrac{1}{1+t}$.

Integrating factor: $\mu(t) = e^{\int \frac{dt}{1+t}} = e^{\ln(1+t)} = (1+t)$

Multiply both sides of ODE by $\mu(t)$:

$$\frac{d}{dt}\left[(1+t)v(t)\right] = -2t^2 - 2t^3$$

Integrate wrt t:

$$(1+t)v(t) = -\frac{2}{3}t^3 - \frac{1}{2}t^4 + C$$

$$\Rightarrow v(t) = \frac{-\frac{2}{3}t^3 - \frac{1}{2}t^4 + C}{1+t}$$

The general solution of the ODE is:

$$y(t) = \frac{1}{v(t)} = \frac{1+t}{-\frac{2}{3}t^3 - \frac{1}{2}t^4 + C}$$