

(b)(i) $A=1, B=-1, C=6, D=0 \Rightarrow \frac{d^2y}{dt^2} - \frac{dy}{dt} = 6y$

$\Rightarrow \frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y = 0$

A homogeneous linear 2nd-order ODE with constant coefficients. Solve by substituting $y(t) = e^{rt}$.

The characteristic equation is: $r^2 - r - 6 = 0$

$r_{1,2} = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2} \quad r_1 = 3 \quad r_2 = -2$

Two distinct real roots.

\Rightarrow The general solution is: $y(t) = C_1 e^{3t} + C_2 e^{-2t}$

(ii) Use IC to find C_1, C_2 .

$y(0) = 2 = C_1 + C_2$

$y'(t) = 3C_1 e^{3t} - 2C_2 e^{-2t} \Rightarrow y'(0) = 3 = 3C_1 - 2C_2$

$3C_1 - 2(2 - C_1) = 3 \Rightarrow 5C_1 = 7 \Rightarrow C_1 = \frac{7}{5}$

$C_2 = 2 - C_1 = \frac{3}{5}$

The solution to the IVP is:

$y(t) = \frac{7}{5} e^{3t} + \frac{3}{5} e^{-2t}$