Therefore: $\frac{dv}{dt} + v = -2e^{2t}$

This is a linear ODE $\rightarrow$ solve by integrating factor.

$\mu(t) = e^{\int dt} = e^t$

Multiply both sides by e^t:

$$\frac{d}{dt}\left[e^t v\right] = -2e^{3t}$$

$$e^t v(t) = -\frac{2}{3}e^{3t} + C \quad \Rightarrow \quad v(t) = Ce^{-t} - \frac{2}{3}e^{3t}$$

Solve for $y(t)$: $y(t) = \frac{1}{v(t)} = \frac{1}{Ce^{-t} - \frac{2}{3}e^{2t}}$

This is the general solution of the ODE.

{ It can also be expressed as:

$$y(t) = \frac{3e^t}{3C - 2e^{3t}} = \frac{3e^t}{K - 2e^{3t}}$$ where $K = 3C$ }

(ii) Use IC to find C.

$$y(0) = \frac{1}{2} = \frac{1}{C - \frac{2}{3}} \Rightarrow C - \frac{2}{3} = \frac{1}{\left(\frac{1}{2}\right)} = 2 \Rightarrow C = \frac{8}{3}$$

$$y(t) = \frac{1}{\frac{8}{3}e^{-t} - \frac{2}{3}e^{2t}}$$ is the solution to the IVP.

$$\left\{ y(t) = \frac{3e^t}{8 - 2e^{3t}} \right\}$$