MTH306Y Spring 2011 solution to midterm exam

1. (a) $K=6$ (growth rate) $M=6$ (carrying capacity)

$h(p)=4+p$ (harvesting function)

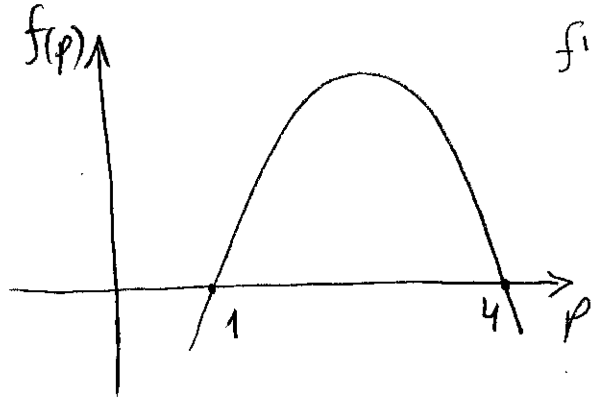
$$\frac{dp}{dt} = \underbrace{kp\left(1-\frac{p}{M}\right)}_{\text{logistic model part}} - \underbrace{h(p)}_{\text{harvesting part}} \Rightarrow \frac{dp}{dt} = 6p\left(1-\frac{p}{6}\right) - (4+p)$$

$$\Rightarrow \frac{dp}{dt} = 5p - p^2 - 4$$

(b) Write $\frac{dp}{dt} = f(p)$ where $f(p) = 5p - p^2 - 4$.

Equilibrium points: $f(p) = 5p - p^2 - 4 = 0 \Rightarrow p^2 - 5p + 4 = 0$

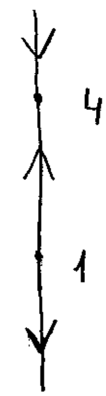
$$p_{1,2} = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} \qquad \underline{p_1 = 4} \quad \underline{p_2 = 1}$$

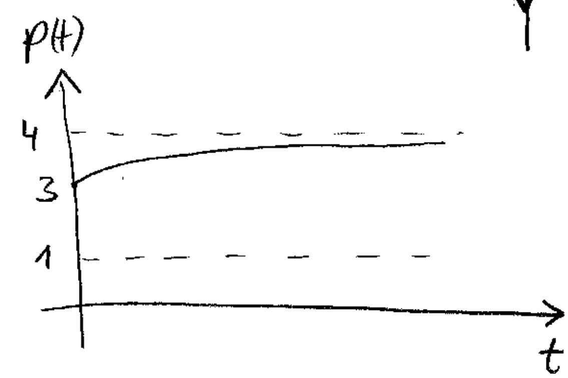


$f'(p) = 5 - 2p$

$f'(1) = 3 > 0 \Rightarrow p_2 = 1$ a source

$f'(4) = -3 < 0 \Rightarrow p_1 = 4$ a sink





(c) For $p(0)=3$ $f(p) > 0$

$\Rightarrow$ p increases and approaches 4 as $t \to \infty$.

(d) The new model is $\frac{dp}{dt} = 6p\left(1-\frac{p}{6}\right) - (8+p)$

$$\frac{dp}{dt} = 5p - p^2 - 8$$