MTH306Y **MIDTERM EXAM** **Name:_______________**

03/10/2011

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Read instructions carefully. Use proper notation and show all work for full credit. Answer both 1 and 2.

1. [50pts]

In the absence of harvesting the population P (in thousands) of a certain fish satisfies the logistic model with growth rate 6 and carrying capacity 6. Suppose that the fish population is harvested, and that the number of fish harvested per unit time is the following linear function of the population: h(P)=4+P.

(a) Obtain the ODE that describes the time evolution of fish population with harvesting. Explain your answer. [6pts]

(b) Calculate and classify the equilibrium points and draw the corresponding phase diagram. [20pts]

(c) Sketch the solution corresponding to the initial condition P(0)=3. What happens to the fish population after a long time in this case? [6pts]

(d) How would your answers to (a) and (b) change, if h(P)=8+P? Show all calculations. If P(0)=3, what happens to the fish population? [18pts]

2. [50pts]

Consider the following ODE

$$A\frac{d^2y}{dt^2} + B\frac{dy}{dt} = Cy + De^{2t}y^2$$

where A, B, C, and D are constants.

(a) Assume that A=0, B=1, C=1, and D=2.

(i) Find the general solution of the ODE in this case. [20pts]

(ii) Find the solution satisfying the initial condition y(0)=1/2. [5pts]

(b) Assume that A=1, B=-1, C=6, and D=0.

(i) Find the general solution of the ODE in this case. [15pts]

(ii) Find the solution satisfying the initial condition y(0)=2, y'(0)=3. [10pts]