(ii) $y(0) = 1 = \tan\left(\frac{2}{3} + k\right)$

$\frac{2}{3} + k = \arctan(1) = \frac{\pi}{4} \quad \Leftrightarrow \quad k = \frac{\pi}{4} - \frac{2}{3}$

The solution of the IVP is

$y(t) = \tan\left[\frac{2}{3}\left(e^{3t} - 1\right) + \frac{\pi}{4}\right]$

(b)(i) For $A = -2$, $B = 4$, and $C = 0$ we obtain:

$\frac{dy}{dt} + 2y = 4e^{3t}y^2$

This is a Bernoulli ODE with $n = 2$, $p(t) = 2$, $q(t) = 4e^{3t}$.

Use the substitution $v = y^{1-n} = y^{-1} = \frac{1}{y}$.

This leads to: $\frac{dv}{dt} + (1-n)p(t)v = (1-n)q(t)$

So the ODE for v is $\frac{dv}{dt} - 2v = -4e^{3t}$

A linear ODE $\rightarrow$ solve by integrating factor.

$\mu(t) = e^{-\int 2dt} = e^{-2t}$

Multiply both sides by e^{-2t}.

$\frac{d}{dt}\left[e^{-2t}v\right] = -4e^{t}$