{ This part is NOT required from the students.

$A\cos(2t-\alpha) = A\cos(2t)\cos(\alpha) + A\sin(2t)\sin(\alpha) = \frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t)$

$\Rightarrow A\cos\alpha = \frac{3}{13} \qquad A\sin\alpha = \frac{2}{13}$

$\Rightarrow A^2 = \frac{3^2}{13^2} + \frac{2^2}{13^2} = \frac{1}{13} \Rightarrow A = \frac{1}{\sqrt{13}}$

$\tan\alpha = \frac{\frac{2}{13}}{\frac{3}{13}} = \frac{2}{3}$ }

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(3) $\frac{dy}{dt} = Ay + Be^{3t}y^2 + Ce^{3t}$

(a)(i) For $A=0$, $B=2$, $C=2$ we obtain:

$\frac{dy}{dt} = 2e^{3t}(y^2+1)$

This is a separable ODE ⇒ can solve by separation of variables.

$\int \frac{dy}{y^2+1} = 2\int e^{3t}dt$

$\arctan(y) = \frac{2}{3}e^{3t} + K$

The general solution is $y(t) = \tan\left(\frac{2}{3}e^{3t} + K\right)$