So the general solution is:

$$y(t) = C_1 e^{-t}\cos(3t) + C_2 e^{-t}\sin(3t) + \frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t)$$

(b) Use IC to find C_1, C_2.

$$y(0) = -1 = C_1 + \frac{3}{13} \quad \Rightarrow \quad C_1 = \frac{-16}{13}$$

$$y'(t) = -C_1 e^{-t}\cos(3t) - 3C_1 e^{-t}\sin(3t) - C_2 e^{-t}\sin(3t) + 3C_2 e^{-t}\cos(3t)$$
$$-\frac{6}{13}\sin(2t) + \frac{4}{13}\cos(2t)$$

$$y'(0) = 2 = -C_1 + 3C_2 + \frac{4}{13}$$

$$\Rightarrow \quad 2 = \frac{16}{13} + \frac{4}{13} + 3C_2 \quad \Rightarrow \quad 3C_2 = \frac{6}{13} \quad \Rightarrow \quad C_2 = \frac{2}{13}$$

The solution of the IVP is:

$$y(t) = -\frac{16}{13} e^{-t}\cos(3t) + \frac{2}{13} e^{-t}\sin(3t) + \frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t)$$

(c) Notice that $y_h(t) \to 0$ as $t \to \infty$ because of the e^{-t} factors. Therefore, the large-t behavior of the solution is:

$$y(t) \simeq y_p(t) = \frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t)$$

This can be expressed as: $y(t) = A\cos(wt - \alpha)$ where

$$A = \left[\left(\frac{3}{13}\right)^2 + \left(\frac{2}{13}\right)^2\right]^{1/2} = \frac{1}{\sqrt{13}} \qquad \tan\alpha = \frac{\frac{2}{13}}{\frac{3}{13}} = \frac{2}{3} \qquad w = 2.$$