Therefore: $y_h(t) = c_1 e^{-t}\cos(3t) + c_2 e^{-t}\sin(3t)$.

Find $y_p(t)$ (a particular solution to the non-h. ODE)

Look for a solution in the form:

$$y_p(t) = A\cos(2t) + B\sin(2t)$$

$$y_p'(t) = -2A\sin(2t) + 2B\cos(2t)$$

$$y_p''(t) = -4A\cos(2t) - 4B\sin(2t)$$

Substitute into ODE:

$$-4A\cos(2t) - 4B\sin(2t) - 4A\sin(2t) + 4B\cos(2t) + 10A\cos(2t) + 10B\sin(2t) = 2\cos(2t)$$

Rearrange terms containing $\cos(2t)$, $\sin(2t)$:

$$(6A+4B)\cos(2t) + (-4A+6B)\sin(2t) = 2\cos(2t)$$

$y_p(t)$ is a solution provided that A, B satisfy:

$$\left.\begin{array}{l} 6A+4B=2 \\ -4A+6B=0 \end{array}\right\} \Leftrightarrow \left.\begin{array}{l} 9B+4B=2 \\ A=\frac{3}{2}B \end{array}\right\} \Leftrightarrow \begin{array}{l} B=\frac{2}{13} \\ A=\frac{3}{13} \end{array}$$

$$\Rightarrow \quad y_p(t) = \frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t)$$

The general solution: $y(t) = y_h(t) + y_p(t)$