Therefore, the solution of the IVP is:

$$X(t) = \begin{pmatrix} -\frac{8}{5}e^{2t} + \frac{18}{5}e^{-3t} \\ \frac{24}{5}e^{2t} - \frac{9}{5}e^{-3t} \end{pmatrix}$$

(c) Characteristic equation: $\lambda^2 + \lambda + 6\lambda = 0$ (*)

Corresponding h. 2nd-order linear ODE with constant coefficients: $\frac{d^2x}{dt^2} + \frac{dx}{dt} - 6x = 0$.

The roots of the char. equation are $r_1 = 2$, $r_2 = -3$.

Since it has 2 distinct real roots, the general solution is:

$$x(t) = c_1 e^{2t} + c_2 e^{-3t}.$$

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(2). (a) Find general solution of $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = 2\cos(2t)$.

This is a non-h. 2nd-order linear ODE with const. coefficients.

Find y_h (general solution of h. ODE)

Char. equation $r^2 + 2r + 10 = 0 \Rightarrow r_{1,2} = \frac{-2 \pm \sqrt{4-40}}{2}$

$$r_{1,2} = \frac{-2 \pm 6i}{2} = -1 \pm 3i$$

Two complex roots with $Re\{r_1\} = -1$, $Im\{r_1\} = 3$.