Find e. vector for $\lambda_2 = -3$. Solve $A\bar{V}_2 = -3\bar{V}_2$.

$$\left.\begin{aligned} -4x - 2y &= -3x \\ 3x + 3y &= -3y \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} -2y &= x \\ 6y &= -3x \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} y &= -\tfrac{1}{2}x \\ y &= -\tfrac{1}{2}x \end{aligned}\right\}$$

Any vector on the line $y = -\frac{1}{2}x$ is an e. vector.

Choose $x = 2 \Rightarrow y = -1$.

$\bar{V}_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ is an e. vector for $\lambda_2 = -3$.

The corresponding nontrivial solution is $\bar{V}_2 e^{\lambda_2 t} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^{-3t}$.

Since A has two distinct real roots, the general solution is a linear combination of $\bar{V}_1 e^{\lambda_1 t}, \bar{V}_2 e^{\lambda_2 t}$:

$$\bar{X}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -3 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^{-3t}$$

$$\bar{X}(t) = \begin{pmatrix} C_1 e^{2t} + 2C_2 e^{-3t} \\ -3C_1 e^{2t} - C_2 e^{-3t} \end{pmatrix}$$

(b) Find the solution satisfying $x(0) = 2$, $y(0) = 3$.

$$\left.\begin{aligned} x(0) = 2 &= C_1 + 2C_2 \\ y(0) = 3 &= -3C_1 - C_2 \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} C_1 &= 2 - 2C_2 \\ 3 &= -6 + 6C_2 - C_2 \end{aligned}\right\} \Leftrightarrow$$

$$\left.\begin{aligned} C_1 &= 2 - 2C_2 \\ 5C_2 &= 9 \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} C_1 &= 2 - \tfrac{18}{5} = -\tfrac{8}{5} \\ C_2 &= \tfrac{9}{5} \end{aligned}\right\}$$