mth 306 spring 2011 solution to final exam

(1) $\frac{d\underline{X}}{dt} = \begin{pmatrix} -4 & -2 \\ 3 & 3 \end{pmatrix} \underline{X} \qquad \underline{X} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$

(a) Find the general solution.

$A = \begin{pmatrix} -4 & -2 \\ 3 & 3 \end{pmatrix}$

Calculate e. values of A.

$$\det(A - \lambda I) = \det \begin{pmatrix} -4-\lambda & -2 \\ 3 & 3-\lambda \end{pmatrix} = 0$$

$$(-4-\lambda)(3-\lambda) + 6 = 0 \quad \Leftrightarrow \quad (\lambda+4)(\lambda-3) + 6 = 0$$

$$\Leftrightarrow \quad \lambda^2 + \lambda - 12 + 6 = 0 \quad \Leftrightarrow \quad \lambda^2 + \lambda - 6 = 0$$

$$\lambda_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} \qquad \lambda_1 = 2 \qquad \lambda_2 = -3$$

Find e. vector for $\lambda_1 = 2$. Solve $A\underline{V}_1 = 2\underline{V}_1$

$$\left.\begin{aligned} -4x - 2y &= 2x \\ 3x + 3y &= 2y \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} -2y &= 6x \\ y &= -3x \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} y &= -3x \\ y &= -3x \end{aligned}\right\}$$

Any vector on the line $y = -3x$ is an e. vector.

chouse $x = 1 \Rightarrow y = -3$.

$\underline{V}_1 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ is an e. vector for $\lambda_1 = 2$.

The corresponding nontrivial solution is $\underline{V}_1 e^{\lambda_1 t} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} e^{2t}$.