MTH306Y **FINAL EXAM** **Name:**

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Instructor: Avner Peleg

Read instructions carefully. For full credit, show all work and explain the main steps in your calculations. Answer all three questions.

1. [33pts]

Consider the following homogeneous system of first order linear ODEs with constant coefficients:

$$\frac{d\boldsymbol{X}}{dt} = \begin{pmatrix} -4 & -2 \\ 3 & 3 \end{pmatrix} \boldsymbol{X} \quad (1)$$

(a) Find the general solution of this system. [18pts]

(b) Find the solution satisfying the initial condition $x(0)=2$, $y(0)=3$. [10pts]

(c) Find the homogeneous second order linear ODE with constant coefficients that has the same characteristic equation as the coefficient matrix of the system (1). Find the general solution of this second order linear ODE. [5pts]

2. [33pts]

(a) Find the general solution of the following ODE: [18pts]

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = 2\cos(2t)$$

(b) Find the solution satisfying the initial condition y(0)=-1, y'(0)=2. [7pts]

(c) Show that the large-t behavior of the solution obtained in (a) can be written as: $y(t)=A\cos(\omega t-\alpha)$ and calculate A, ω, and tan(α). [8pts]

3. [34pts]

Consider the following first order ODE

$$\frac{dy}{dt} = Ay + Be^{3t}y^2 + Ce^{3t}$$

(a) Assume that A=0, B=2, and C=2.

(i) Find the general solution of the ODE in this case. [10pts]

(ii) Find the solution satisfying the initial condition y(0)=1. [5pts]

(b) Assume that A=-2, B=4, and C=0.

(i) Find the general solution of the ODE in this case. [14pts]

(ii) Find the solution satisfying the initial condition y(0)=1. [5pts]